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ABSTRACT

Possible applications of the principle of similarity to fluid mechanics is described and illustrated. In correlating thermophysical properties of fluids, the similarity principle transcends the traditional corresponding states principle. In fluid mechanics the similarity principle is useful in correlating flow processes that can be modeled adequately with one independent variable (i.e., one-dimensional flows). In this paper we explore the concept of transforming the conservation equations by COMBINING similarity principles for thermophysical properties with those for fluid flow. We illustrate the usefulness of the procedure by applying such a transformation to calculate two phase critical mass flow through a nozzle.

KEYWORDS

Fluid mechanics; principles of similarity; thermophysical properties; corresponding states; thermodynamic critical point; two phase choked flows.

INTRODUCTION

Many dynamic processes can be modeled in terms of one physical coordinate dimension and the success of such modeling is well known. However, in doing so one must realize that not all boundary conditions associated with higher dimension coordinate spaces can be satisfied; implying the use of a set of suitable constraints, a priori, which "average" the dynamic character of the flow field. For example, in fluid mechanics the representative velocity, heat flux, and friction factor can be written as

$$\bar{u} = \frac{1}{A_1 t_0} \int \vec{u}(A_1, t) \cdot \vec{n} \, dA \, dt; \quad \bar{q} = \frac{1}{A_1 t_0} \int \vec{q}(A_1, t) \cdot \vec{n} \, dA \, dt$$

$$\bar{f}_f = \frac{1}{A_1 t_0} \int f_f(A_1, t) \cdot dA \, dt$$

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where A_1 is a surface and t_0 a specific time interval. In order to formulate a set of constraints, one postulates the existence of principles of similarity between the thermodynamic parameters and the conservation equations, such as constant or power law dependent fluid properties, or coordinates which behave in such a way as to satisfy separation of variables. Often such relations imply a "slug flow" velocity profile, or constant heat flux or an average friction factor based on the inlet and exit pressure differences. Under these conditions, only a limited number of boundary conditions can possibly be satisfied.

A family of functions of a number of physical variables is said to satisfy a similarity principle if the function can be made identical by reducing the functions and variables in terms of suitably chosen scale factors. Thus when such scale factors can be found as units to nondimensionalize (normalize) other variables, the profiles plotted in these dimensionless coordinates become congruent. For instance, in fluid mechanics when the momentum equation, cited below, is made dimensionless using the fluid velocity (u), density (ρ), viscosity (η) and a characteristic length (l), it follows that two flows become similar when their Reynolds numbers are equal

$$Re_1 = \frac{\rho_1 u_1 l_1}{\eta_1} = \frac{\rho_2 u_2 l_2}{\eta_2} = Re_2$$

and this is known as Reynolds principle of similarity. For thermophysical properties, van der Waals' principle of corresponding states assumes that property surfaces become similar when the variables describing these surfaces are properly scaled, in terms of the critical point parameters,

$$P_{R_1}(T_{R_1}, V_{R_1}) = P_{R_2}(T_{R_2}, V_{R_2})$$

where $P_R = P/P_c$, $T_R = T/T_c$ and $V_R = V/V_c$. And in mathematics, similarity solutions are based on suitable scale factors which stretch the coordinates and reduce the governing partial differential equations to ordinary ones. When confronted with a complex problem, it is often quite useful to determine the exact solution subject to restricted geometric conditions similar to those of the original problem. Boundary layer theory is rich in such an approach where most similarity solutions are also one dimensional (Bird and colleagues, 1960; Gebeci and Smith, 1974; Hansen, 1964; Hinze, 1975; Moore, 1964; Schlichting, 1968).

The reason for similarity and/or one dimensional modeling can be readily understood by considering the conservation equations of fluid mechanics (Bird and colleagues, 1960; Gebeci and Smith, 1974; Hinze, 1975; Schlichting, 1968; Zeleznik, 1976).

$$\text{Continuity: } \frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0$$

$$\text{Momentum: } \frac{\partial \rho u_i}{\partial t} + \frac{\partial}{\partial x_j} \rho u_i u_j = \frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \rho g_i$$

$$\text{Energy: } \frac{\partial \rho H_o}{\partial t} + \frac{\partial}{\partial x_j} \rho u_j H_o = \frac{\partial}{\partial x_j} (u_i \tau_{ij} - q_j) + \frac{\partial p}{\partial t}$$

$$\text{State: } P = P(\rho, H, x_\alpha)$$

where

$$H_0 = H + \frac{1}{2} u_i u_i$$

These equations are difficult to solve without some simplifying assumptions, even for potential flows. With added complexities, such as diabatic flows with real fluid properties and severe gradients, the mass and energy fluxes, τ_{ij} and q_j respectively, become most difficult to determine as BOTH τ_{ij} and q_j are related and yet constrained by some boundary. Much of our information for these related constraints is constructed from engineering data for the fluids water and air. Still our knowledge is limited and we are forced to make simplifying assumptions.

For the cases where tractable analytical and/or numerical treatments are available, regardless of simplifications, we must have accurate, consistent thermophysical property information to transform these solutions into physical reality. Sometimes this information is limited or unavailable, yet the success of the entire transformation depends on its accuracy. So we need to develop methods to predict thermophysical properties and techniques to relate these results to an experimental body of data for fluid α to a similar experiment with fluid β without essentially reproducing the entire experiment.

Following van der Waals' principle of similarity, many thermophysical properties for several species seem to be correlated in the sense that one can introduce suitable scale factors in the dependent and independent variables so that properties so reduced are congruent to some universal curve (i.e., become universal). This applies to normal as well as near critical behavior. But since flow and heat transfer behavior depend on these properties, it seems reasonable to expect that for similar geometric arrangements it should be possible to describe flow and heat transfer in such a way as to be, more or less, congruent to some universal curves.

In general then, one would like to develop solutions which are independent of BOTH the physical nature of the coordinates and the thermodynamic character of the working fluid. As an attempt toward accomplishing this, we shall first review some similarity principles for the thermophysical properties of fluids in section A, with emphasis on the critical region in section B and cite some related computational programs in section C; second, we shall provide some examples of physical processes which have been related using similarity principles, in section D; and third, in section E we use these similarity principles to demonstrate that two-phase choked flows for a simple fluid α can be computed using a simple fluid β independent of the thermodynamic character of the other fluids α and β thereby demonstrating the process independence of BOTH coordinates and working fluid; our conclusions are summarized in section F.

A. THE PRINCIPLE OF SIMILARITY IN DESCRIBING THERMOPHYSICAL PROPERTIES OF FLUIDS

A1. Background

Most engineering texts on thermodynamics treat the subject of corresponding states to some degree for both single and multicomponent fluids, while some consider a possible formulation of corresponding states for transport properties. However, the actual practice of predicting such fluid properties as a function of the thermodynamic state variables requires a great deal of computing capability and accurate $PVTx_\alpha$ data for developing fundamental equations. In this section, we will mention some approaches.

A2. Thermophysical Properties of One-Component Fluids

A2.1 Thermodynamic Properties

As an example of the traditional corresponding states methods, consider van der Waals' equation of state describing fluid α .

$$P = \frac{RT}{v-b} - \frac{a}{v^2} \quad (\text{A.1})$$

In reduced form it becomes,

$$P_R = \frac{T_R}{Z_c (V_R - \Omega_b/Z_c)} - \frac{\Omega_a}{Z_c^2 V_R^2} \quad (\text{A.2})$$

where

$$Z_c = \frac{1}{3}; \Omega_b = \frac{1}{8}; \Omega_a = \frac{27}{64}; P_R = \frac{P}{P_c}; V_R = \frac{V}{V_c}; T_R = \frac{T}{T_c} \quad (\text{A.3})$$

and in this form it not only describes fluid α but fluid β as well. This approach is correct for any system of molecules whose potential can be specified by two system-dependent parameters only.

Mollerup (1975 and 1977) proposes to relate the properties of two pure substances by the equations¹:

$$Z_\alpha(V, T) = Z_o\left(\frac{V}{h}, \frac{T}{f}\right) \quad (\text{A.4})$$

$$G_\alpha(V, T) = G_o\left(\frac{V}{h}, \frac{T}{f}\right) - RT \ln h \quad (\text{A.5})$$

where o denotes the reference fluid and the properties are termed configurational because they evolve from the configurational part of the partition function and are conformal in that they have similar molecular potentials and obey the principle of corresponding states; for example, both are adequately described by a two parameter Lennard-Jones type potential. The functions f and h are dependent on the ratio of the critical temperature and volume

$$f = \left(\frac{T_c}{T_o}\right)^\theta \quad h = \left(\frac{V_c}{V_o}\right)^\phi \quad (\text{A.6})$$

while θ and ϕ are shape factor functions that depend on the accentric factors of fluid α and fluid o and that are weakly dependent on temperature and volume. (See Leach, 1966, 1967; and Eqs. (E.5) and (E.6)).

Mollerup (1978) has used the Peng-Robinson equation to predict properties of the quantum fluids hydrogen and helium using accentric factors of approximately -0.2187 and -0.3897 for hydrogen and helium respectively along with the extended corre-

¹Note that the related forms become

$$P_o \rightarrow \frac{Ph}{f}; V_o \rightarrow \frac{V}{h}; \rho_o \rightarrow \rho h; T_o \rightarrow \frac{T}{f}$$

sponding states theory. Mollerup finds that the shape factor can be represented by

$$\theta = \frac{1 + m_\alpha + (m_\beta - m_\alpha) \sqrt{T_{R\alpha}}}{1 + m_0} \quad (A.7)$$

for quantum fluids, where m_i is a quadratic function in acentric factor ω_i . He has verified this representation at reduced temperatures up to $T_R \leq 26$ for hydrogen and $T_R \leq 40$ for helium.

A2.2 Transport

Dimensional analysis² and corresponding states can be combined to determine suitable scale factors for transport coefficients.

1. Viscosity data for many fluids can be grouped over a large range in experimental conditions (Dean and Stiel, 1965; Jossi Stiel, and Thodos, 1962; Stiel and Thodos, 1962) using the parameters

$$(\bar{\eta} - \eta_{0,0}) \xi_1 = f(T_R, \rho_R) \quad (A.8)$$

where

$$\xi_1 = \frac{T_c^{1/6}}{P_c^{2/3} M^{1/2}} \quad (A.9)$$

as illustrated in Fig. 1.

2. Thermal conductivity data were grouped using a similar parameter (Misic and Thodos, 1961; Schaefer and Thodos, 1959)

$$(\bar{\lambda} - \lambda_{0,0}) Z_c^5 \xi_2 = f(T_R, \rho_R) \quad (A.10)$$

where

$$\xi_2 = M \xi_1 \quad \text{and} \quad Z_c = \frac{P_c V_c}{RT_c} \quad (A.11)$$

3. Binary diffusion coefficients were grouped using the parameter (Statter and Bird, 1958)

$$D_{12} \xi_3 = f(T_R, P_R) \quad (A.12)$$

where

$$\xi_3 = \frac{P_c^{1/3} M^{1/2}}{T_c^{5/6}} \quad (A.13)$$

²Dynamically similar processes based on the observation that physical laws are independent of the particular system of units employed.

4. Surface tension data can be grouped with the parameter (Brock and Bird, 1955)

$$\sigma \xi_4 = f(T_R) \quad (A.14)$$

where

$$\xi_4^{-1} = P_c^{2/3} T_c^{1/3} \quad (A.15)$$

Hanley (1977) and Rainwater (1977) applied these configuration functions to thermal conductivity and viscosity and proposed that the thermal conductivity and viscosity of pure components could be related as³

$$\lambda_\alpha(\rho, T) = \lambda_o\left(\rho h, \frac{T}{f}\right) F_\lambda \quad (A.16)$$

$$\eta_\alpha(\rho, T) = \eta_o\left(\rho h, \frac{T}{f}\right) F_\eta \quad (A.17)$$

where

$$F_\lambda = \left(\frac{M_o}{M_\alpha} f\right)^{1/2} / h^{2/3} \quad F_\eta = \left(\frac{M_\alpha}{M_o}\right) F_\lambda \quad (A.18)$$

The use of configurational functions f and h along with an accurate reference equation of state greatly enhances the users capability to accurately relate thermophysical properties. It does however have the disadvantage of being more complex and one needs to determine the configuration factors.

More recent representations of the viscosity surface (in terms of density virials with temperature dependent coefficients) are given by Hanley, McCarty and Haynes (1974a) using a corresponding states approach to correlate the results for several simple fluids. The procedure is more accurate but can not be applied to as many different fluids as some more conventional principles which have been successfully applied not only to simple fluids but to long chain hydrocarbons and polar molecules (Dean and Stiel, 1965; Jossi, Stiel, and Thodos, 1962; Stiel and Thodos, 1962).

A3. Thermophysical Properties of Multicomponent Fluids (Fluid Mixtures)

A3.1 Perfect Gases

Gordon and McBride (1971), Zeleznik and Gordon (1968), and Svhela (1962) have formulated a program which calculates the chemical equilibrium composition and mixing properties of multicomponent fluids. The program can handle fuels and oxidants constructed from 15 different chemical elements. There are some 600 different products of combustion whose data are stored in the computer program, and any given

³Using the related forms of Eq. (A.4)-footnote, we can relate traditional parameters and the extended corresponding states parameters

$$\xi_1 = F_\eta; \xi_2 = \left(\frac{M_o}{M_\alpha} f\right)^{1/2} h^{1/3}; \xi_4^{-1} = \frac{f}{h^{2/3}}$$

computation can handle any 200 of them. The program includes solid and liquid condensed phases and ionization. Bittker and Scullin (1972) have a code for reaction kinetics. These programs represent a major contribution to combustion computations and are based on free energy functions. The thermodynamic and transport property contributions of Kestin (1979) and collaborators in formulating accurate representations of the thermodynamic and transport properties of pure component fluids and fluid mixtures are well known. Most recently they have developed a formula for the transport properties of natural gas (Abe and Kestin, 1979).

A3.2 Extended Principle-Multicomponent Fluids

A3.2.1 Thermodynamic Properties

Mollerup (1975, 1977, 1978) used an extended corresponding states principle to correlate and predict properties of some fluid mixtures with considerable accuracy. In this procedure, a reference substance is chosen, usually the most dominant of the mixture with the most accurate PVT surface description.

$$Z_m(V, T, x_\alpha) = Z_o\left(\frac{V}{h}, \frac{T}{f}\right) \quad (A.19)$$

$$G_m(V, T, x_\alpha) = G_o\left(\frac{V}{h}, \frac{T}{f}\right) + RT \left(\sum_{\alpha} x_{\alpha} \ln x_{\alpha} - \ln h \right) \quad (A.20)$$

The form is similar to that for pure substances except that we need to account for the energy of mixing and introduce some "mixing rules" which for our case are van der Waals' two component rules. While there are certainly other potentially applicable versions, these appear to be the simplest and most effective. We must now redefine the functions f and h :

$$h = \sum_{\alpha} \sum_{\beta} x_{\alpha} x_{\beta} h_{\alpha\beta} \quad (A.21)$$

$$fh = \sum_{\alpha} \sum_{\beta} x_{\alpha} x_{\beta} h_{\alpha\beta} f_{\alpha\beta} \quad (A.22)$$

where

$$f_{\alpha\beta} = \xi_{\alpha\beta} (f_{\alpha\alpha} f_{\beta\beta})^{1/2} \quad (A.23)$$

$$h_{\alpha\beta} = \varphi_{\alpha\beta} \left(\frac{h_{\alpha\alpha}^{1/3} + h_{\beta\beta}^{1/3}}{2} \right)^3 \quad (A.24)$$

and where $\xi_{\alpha\beta}$ and $\varphi_{\alpha\beta}$ are binary interaction parameters. While the procedure is complex, Mollerup has already determined many interaction parameters of practical interest, for LNG, and has written a computer program to predict these properties.

A3.2.2 Transport Properties

Hanley and Rainwater (1977) applies these configuration functions and proposed that thermal conductivity and viscosity of mixtures could be related in a manner similar to pure components, Eqs. (A.16) and (A.17),

$$\lambda(\rho, T, x_\alpha) = \lambda_o \left(\rho h, \frac{T}{f} \right) F_\lambda \quad (A.25)$$

$$\eta(\rho, T, x_\alpha) = \eta_o \left(\rho h, \frac{T}{f} \right) F_\eta \quad (A.26)$$

where f and h are defined as above, but F_λ , F_η are redefined.⁴ For the thermal conductivity, Hanley proposed an expression based on the theory of Enskog

$$x_{\alpha,0}^\lambda = Q_{\alpha,0} G_{\alpha,0}^\lambda \quad (A.27)$$

$$x_{\alpha,0}^\eta = Q_{\alpha,0} G_{\alpha,0}^\eta \quad (A.28)$$

where

$$Q_{\alpha,0} = \left[\frac{(b\rho)_\alpha}{(b\rho)_o} \right] \left[1 - 1/\exp(\rho_c/\rho) \right]^3 \quad (A.29)$$

$$G_{\alpha,0}^{\left[\right]} = \frac{\kappa_\alpha^{\left[\right]}}{\kappa_o^{\left[\right]}} \quad (A.30)$$

$$\kappa^\lambda \equiv \frac{1}{\rho b X} + 1.2 + 0.775 \rho b X \quad (A.31)$$

$$\kappa^\eta \equiv \frac{1}{\rho b X} + 0.8 + 0.761 \rho b X \quad (A.32)$$

F_λ , F_η now can be written as

$$F_\lambda = \left[\left(\frac{M_o}{M_\alpha} f \right)^{1/2} / h^{2/3} \right] x_{\alpha,0}^\lambda \quad (A.33)$$

$$F_\eta = \frac{M_\alpha}{M_o} \frac{G_{\alpha,0}^\eta}{G_{\alpha,0}^\lambda} F_\lambda \quad (A.34)$$

⁴It should follow that Prandtl number could be related using these principles.

Pr = mass diffusion/thermal diffusion = ν/α (for const. properties, $Pr = c_p(\eta/\lambda)$)

$$Pr(\rho, T)_\alpha = Pr(\rho h, T/f)_o$$

or

$$[Pr(\rho, T) - Pr(T)]_\alpha = (\Delta Pr)_o$$

B. SIMILARITY OF THERMOPHYSICAL PROPERTIES OF FLUIDS IN THE CRITICAL REGION

B.1 Background

Many thermophysical properties exhibit anomalous behavior in the vicinity of the critical point. For instance, the isothermal compressibility, the thermal expansion coefficient and the specific heat of fluids all diverge at the critical point. Critical enhancement effects are also encountered in the behavior of the thermal conductivity and the viscosity near the critical point.

It has always been difficult to formulate correlating equations that accurately incorporate the singular behavior of the various thermophysical properties near the critical point. However, in recent years considerable progress has been made in the art of developing representative equations for the thermodynamic and transport properties of fluids in the critical region.

The critical point is a point of marginal thermodynamic stability. In the vicinity of the critical point, large-scale density fluctuations are present in the fluid. The spatial extent of these fluctuations may be characterized by a coherence or correlation length ξ . Near the critical point, this correlation length becomes much larger than the intermolecular interaction range. One can imagine that the critical enhancement effects are caused by the static and dynamic behavior of a system of clusters. Different states in the vicinity of the critical point correspond to different cluster sizes and the thermodynamic states can be interrelated by scaling laws. Moreover, the properties of a system of clusters become independent of the nature of the individual molecules. This principle is currently known as critical-point universality (A. Sengers and J. V. Sengers, 1977).

Recent developments have demonstrated that these principles become rigorously valid only in an asymptotic small region around the critical point, a region too small to be of interest in engineering applications. However, using some semi-empirical adaptations, the approach can nevertheless be fruitfully used to formulate representations that can be used in practice in a larger range around the critical point (J. V. Sengers and J. M. H. Sengers, 1978). In this section we consider these somewhat empirical generalizations of the principles of scaling and universality for representing thermophysical properties of fluids in the critical region.

B2. Thermophysical Properties

B2.1 Power Laws

Let F be the Helmholtz free energy, S the entropy, μ the chemical potential per unit mass and C_V the heat capacity at constant volume. We also find it convenient to introduce a symmetrized isothermal compressibility defined as

$$\chi_T \equiv \left(\frac{\partial \rho}{\partial \mu} \right)_T = \rho^2 k_T \quad (\text{B.1})$$

The thermodynamic properties are made dimensionless by expressing them in terms of the critical temperature T_c , the critical density ρ_c , and the critical pressure p_c .

$$\left. \begin{aligned} T^* &= T/T_c, \rho^* = \rho/\rho_c, P^* = P/P_c, F^* = F/P_c V, \\ \mu^* &= \mu P_c/P_c, \chi_T^* = \chi_T P_c/\rho_c^2, S^* = S T_c/P_c V, C_V^* = C_V T_c/P_c V \end{aligned} \right\} \quad (B.2)$$

Note that the reduced extensive properties F^* , S^* and C_V^* are all taken per unit volume rather than unit mass. The reason is that the singular part of the extensive thermodynamic properties per unit volume appear to be symmetric functions (Vincentini-Missoni, Sengers, and Green, 1969; J. M. H. Sengers, 1974) of $(\rho - \rho_c)$. As a consequence, χ_T is also a symmetric function of $\rho - \rho_c$, while the compressibility $k_T \equiv \rho^{-1}(\partial \rho / \partial P)_T$ is not (J. V. Sengers, 1973). We also define the reduced differences

$$\left. \begin{aligned} \Delta T^* &= (T - T_c)/T_c \\ \Delta \rho^* &= (\rho - \rho_c)/\rho_c \\ \Delta \mu^* &= [\mu(\rho, T) - \mu(\rho_c, T)] \rho_c / P_c \end{aligned} \right\} \quad (B.3)$$

To represent the singular thermodynamic behavior of fluids in the vicinity of the critical point, one first defines critical power laws (Fisher, 1967). The exponents of these power laws depend on the property considered and the path along which the critical point is approached. The special paths in the $\Delta T^* - \Delta \rho^*$ plane commonly considered are the coexistence curve or phase boundary (CXC), the critical isochore (CIC) $\Delta \rho = 0$ and the critical isotherm (CIT) $\Delta T^* = 0$. These paths are schematically indicated in Fig. 2. The critical power laws defined along these paths are summarized in Table I. The critical exponents of the different thermodynamic properties are not independent. The scaling laws to be discussed below imply the exponent relations

$$\left. \begin{aligned} \alpha &= 2 - \beta(\delta + 1) \\ \gamma &= \beta(\delta - 1) \end{aligned} \right\} \quad (B.4)$$

2.2 Scaling Laws

In order to formulate scaling laws for the critical region, we introduce a scaling variable x defined as

$$x = \Delta T^* / |\Delta \rho^*|^{1/\beta} \quad (B.5)$$

This scaling variable is chosen such that the coexistence boundary corresponds to a constant value of the scaling variable x :

$$x_{CXC} = -x_0, \quad x_0 = B^{-1/\beta} \quad (B.6)$$

Instead of x , it is slightly more convenient to use the variable

$$y = \frac{x + x_0}{x_0} \quad (B.7)$$

Curves of constant y are schematically indicated in Fig. 3. The coexistence curve corresponds to $y = 0$, the critical isotherm to $y = 1$ and the critical isochore to $y = \infty$.

In the critical region the thermodynamic properties are considered as a sum of two types of contributions, namely one type of contribution that represents the singular behavior at the critical point and satisfies scaling laws and another type of contribution that is a regular or "background" contribution. The scaled expressions for a number of thermodynamic functions are summarized in Table 2 (J. V. Sengers and J. M. H. Sengers, 1978). Here the singular behavior of the thermodynamic properties is expressed in terms of two scaling functions $h(y)$ and $a(y)$ which are related by the differential equation (J. M. H. Sengers, Greer, and J. V. Sengers, 1976)

$$\beta h(y) = \beta(\delta + 1)a(y) + (1 - y) \frac{da(y)}{dy} \quad (\text{B.8})$$

The function $h(y)$ is normalized such that $h(1) = 1$.

As an example, we consider the scaling law for the chemical potential

$$\Delta\mu^* = \Delta\rho^* |\Delta\rho^*|^{\delta-1} Dh(y) \quad (\text{B.9})$$

Normally, the chemical potential is a function of two variables, density and temperature. The scaling law (B.9) implies that when the data for $\Delta\mu^*$ are divided by $\Delta\rho^* |\Delta\rho^*|^{\delta-1}$, the data should collapse onto a single curve as a function of the scaling variable y . In Fig. 4 we show such a scaled plot for steam (J. M. H. Sengers, Greer, and J. V. Sengers, 1976).

To use the scaling laws in practice, one needs an explicit expression for the scaling function $h(y)$. An expression proposed for this purpose is the so-called NBS- or MSLG-equation (Sengers, Greer, and Sengers, 1976; Sengers and Sengers, 1977; Vincentini-Missoni, Sengers, and Green, 1969)

$$h(y) = y \left(\frac{1 + Ey^{2\beta}}{1 + E} \right)^{(\gamma-1)/2\beta} \quad (\text{B.10})$$

where E is a constant to be discussed below. The advantage of this expression is that it enables one to calculate the compressibility χ_T^* directly as a function of density and temperature in the critical region. The disadvantage is that it cannot be analytically integrated to yield an explicit expression for $a(y)$ and, hence, for the pressure. This problem is solved by introducing two auxiliary ("parametric") variables, replacing the variables ΔT^* and $\Delta\rho^*$. For a discussion of the parametric equations for the thermodynamic properties in the critical region, the reader is referred to a review published elsewhere (Sengers and Sengers, 1978).

It follows from (B.5) and (B.9), that all critical exponents for the power laws in Table 1 are related to the exponents β and δ . These relations were earlier given in (B.4). The amplitudes B , Γ and Γ' of the critical power laws for the coexistence curve and the compressibility are related to D and x_0 by

$$B = x_0^{-\beta} \quad (\text{B.11a})$$

$$\Gamma = \frac{x_o^\gamma}{D} \left(\frac{1+E}{E} \right)^{(\gamma-1)/2\beta} \quad (\text{B.11b})$$

$$\Gamma' = \frac{\beta x_o^\gamma}{D} (1+E)^{(\gamma-1)/2\beta} \quad (\text{B.11c})$$

2.3 Universality

The principle of universality asserts that the critical exponents and the scaling function $h(y)$ should be universal, that is, the same for all 3-dimensional systems with short range forces. The universal values for the critical exponents of this universality class have been calculated theoretically (Baker, Nickel, and Meiron, 1978; Guillon and Zinn-Justin, 1977). The only system dependent constants in the scaling laws are the coefficients D and x_o which specify the amplitude of the critical isotherm and the coexistence curve, respectively.

It turns out that the principle of universality in its strict sense applies in a small region $\Delta T^* \leq 10^{-4}$ near the critical point which is insignificantly small for engineering applications (Sengers and Sengers, 1977). However, an empirical extension of the universality principle as applied to fluids near the critical point is obtained by introducing effective fluid exponents and an effective value for the constant α in (B.10) (Sengers, Groer, and Sengers, 1976; Sengers and Sengers, 1976; Sengers and Sengers, 1978)

$$\alpha = 0.100, \beta = 0.355, \gamma = 1.19, \delta = 4.352, E = 0.287 \quad (\text{B.12})$$

As an illustration of this practical application of universality, we show in Fig. 5 scaled chemical potential data (Sengers, Greer, and Sengers, 1976; Sengers and Sengers, 1978) for ^3He , ^4He , Xe, CO_2 and H_2O . The data can approximately be represented by a single function of the scaling variable $y = (x + x_o)/x_o$. The data cover a range in temperature and density corresponding to

$$5 \times 10^{-4} \leq |\Delta T^*| \leq 3 \times 10^{-2}, |\Delta \rho^*| \leq 0.25 \quad (\text{B.13})$$

The principle of corresponding states would imply identical behavior when the thermodynamic properties are reduced using the critical parameters T_c , ρ_c , P_c . The fluids included in Fig. 5 do not satisfy corresponding states. However, the universality hypothesis is a generalization of corresponding states, saying that universal critical behavior is obtained using only two additional constants, D and x_o , besides the critical parameters themselves. In Table 3, taken from an earlier review (Sengers and Sengers, 1978), we have listed these parameters for a large number of fluids.

The principle of universality applies to the compressibility χ_T^* and to the singular contributions of the other thermodynamic properties (see Table 2). It may be noted that $(\partial P / \partial T)_\rho$ is finite at the critical point, while the compressibility χ_T diverges strongly. It has been found in some practical applications that already a significant improvement is obtained if χ_T is calculated from the scaling laws even if the equation of state and, hence $(\partial P / \partial T)_\rho$ are represented by more conventional equations.

3. Correlation Length

An important quantity in the critical region is the correlation length ξ which measures the size of the clusters. In practice this correlation length may be estimated from (Sengers and Sengers, 1978)

$$\xi = \xi_0 (\chi_T^* \Gamma)^{\nu/\gamma} \quad (\text{B.14})$$

where Γ is given by (B.11b). The critical exponent ν for the correlation length is related to the thermodynamic exponents β and γ by the hyperscaling relation

$$\nu = (2\beta + \gamma)/3 \quad (\text{B.15})$$

Using the values adopted in (B.12) one finds $\nu = 0.633$. The scale factor ξ_0 for the correlation length can be related to the critical power amplitudes B and Γ by (Sengers and Sengers, 1978)

$$\xi_0 (B^2 P_c / \Gamma k_B T_c)^{1/3} = R_1 \quad (\text{B.16})$$

where R_1 is a universal constant whose value is approximately equal to 0.7. Values of the correlation length scale factor ξ_0 for a number of fluids are also included in Table 3.

4. Transport Properties

4.1 Thermal Conductivity

The thermal conductivity of fluids shows a pronounced critical enhancement in a large range of temperatures and densities around the critical point (Sengers, 1972, 1973). To account for this behavior the total thermal conductivity λ is decomposed as the sum of a normal part $\tilde{\lambda}$ and a critical part $\Delta\lambda$

$$\lambda = \tilde{\lambda} + \Delta\lambda \quad (\text{B.17})$$

The idea is that the normal part $\tilde{\lambda} = \bar{\lambda} + \lambda_{0,0}$ represents the thermal conductivity without the extra contributions due to the critical fluctuations. This part, therefore, can be represented by customary correlating procedures, section section A.

The theory of dynamical critical phenomena predicts that the critical enhancement $\Delta\lambda$ will diverge asymptotically at the critical point as (Hohenberg and Halperin, 1967; Kawasaki, 1970; Sengers, 1973)

$$\Delta\lambda \rightarrow \Delta\lambda_c = \frac{\Lambda}{6\pi\eta\xi} \frac{k_B T^2}{\rho^2} \left(\frac{\partial p}{\partial T} \right)_\rho^2 \chi_T = \frac{\Lambda k_B P_c}{6\pi\eta\xi} \frac{T^{*2}}{\rho^{*2}} \left(\frac{\partial p^*}{\partial T^*} \right)_\rho^2 \chi_T^* \quad (\text{B.18})$$

where k_B is Boltzmann's constant, η the shear viscosity and Λ a universal constant of order unity. Values predicted theoretically for this constant Λ span the range $1.0 \leq \Lambda \leq 1.2$.

An empirical generalization of (B.18) to cover the entire range of temperatures and densities of the thermal conductivity enhancement, is obtained by writing (Hanley, McCarty, and Sengers, 1974; Hanley, Sengers, and Ely, 1976),

$$\Delta\lambda = \Delta\lambda_c f_1 |\Delta T^*, \Delta\rho^*| \quad (\text{B.19})$$

where $\Delta\lambda_c$ is given by (B.18) and where $f(\Delta T^*, \Delta\rho^*)$ is an empirical damping function such that

$$\lim_{\Delta T^* \rightarrow 0, \Delta\rho^* \rightarrow 0} f_1(\Delta T^*, \Delta\rho^*) = 1$$

and

$$\lim_{\Delta T^* \rightarrow \infty, \Delta\rho^* \rightarrow \infty} f_1(\Delta T^*, \Delta\rho^*) = 0$$

An example of a suitable damping function is (Hanley, McCarty, and Sengers, 1974; Hanley, Sengers, and Ely, 1976)

$$f_1(\Delta T^*, \Delta\rho^*) = \exp \left\{ - \left[c_T (\Delta T^*)^2 + c_\rho (\Delta\rho^*)^4 \right] \right\} \quad (\text{B.20})$$

with $c_T = 18.66$ and $c_\rho = 4.25$. Equations of the type given in (B.19) have been used to represent the thermal conductivity of a variety of fluids (Hanley and colleagues, 1974a, 1974b, 1976, 1977a, 1977b) including steam (Basu and Sengers, 1979).

4.2 Viscosity

Theory predicts that the shear viscosity η of fluids also diverges at the critical point (Hohenberg and Halperin, 1967; Ohta and Kawasaki, 1976; Perl and Ferrell, 1972). Hence, we write in analogy to (B.17)

$$\eta = \tilde{\eta} + \Delta\eta \quad (\text{B.21})$$

where $\tilde{\eta} = \bar{\eta} + \eta_{0,0}$ is the normal viscosity and $\Delta\eta$ the critical viscosity enhancement. It turns out that the critical viscosity enhancement is a multiplicative enhancement, that is, $\Delta\eta$ is itself proportional to $\tilde{\eta}$ (Ohta, 1977). Therefore, in practice, we prefer to consider the viscosity ratio $\Delta\eta/\tilde{\eta}$ or $\eta/\tilde{\eta}$ (Basu and Sengers, 1979).

Unlike the critical thermal conductivity enhancement, the critical viscosity enhancement is weak and limited to a small range of temperatures and densities around the critical point (Basu and Sengers, 1979). Hence, the enhancement may be neglected for many applications as was done in the viscosity equation recently adopted by IAPS for steam (Hendricks, McClintock, and Silvestri, 1977; Nagashima, 1977; Schleffer and colleagues, 1978).

A more detailed discussion of the critical viscosity enhancement of fluids near the gas-liquid critical point is presented elsewhere (Basu and Sengers, 1979; Basu, Sengers, and Watson, unpublished). It can be estimated as

$$\left. \begin{aligned} \eta &= \tilde{\eta} (q_1 \xi)^{\phi_1} & \text{for } q_1 \xi \geq 1 \\ \eta &= \tilde{\eta} & \text{for } q_1 \xi \leq 1 \end{aligned} \right\} \quad (\text{B.22})$$

where ϕ_1 is a universal critical exponent and q_1 a parameter depending on the fluid. The effect of the critical enhancement is small and accurate knowledge of

ϕ_1 and q_1 is not needed for practical applications. As a guide we recommended an effective exponent value (Basu and Sengers, 1979)

$$\phi_1 = 8/15\pi^2 \approx 0.054 \quad (\text{B.23})$$

while the parameter q_1 may be estimated from (Basu and Sengers, 1979; Oxtoby and Gelbart, 1974)

$$q_1 \approx \frac{k_B T^2}{8\pi\eta\lambda\rho^2} \left(\frac{\partial P}{\partial T}\right)^2 \frac{x_T}{\xi^2} \approx \frac{k_B P_c}{8\pi\eta\lambda} \frac{\Gamma}{\xi_0^2} \left(\frac{\partial P^*}{\partial T^*}\right)^2_\rho \quad (\text{B.24})$$

while, as "a rule of thumb," $(\partial P^*/\partial T^*)_\rho \approx 6$, see Eq. (D.4).

C. NUMERICAL PLOTTING CODES

Before proceeding, we must caution the reader that accurate representations of the PVT surfaces are a requirement, for a meaningful transformation of any solution to physical reality.

Equations of state (EOS) are available from NBS, IUPAC, HTFS, NASA, Tech. Univ. Munich, etc. and mixtures from Mollerup Tech. Univ. of Denmark, AGA, NBS, NASA and the Petro Chem. Ind., API.⁵

With the advent of the computer and plotting programs, such as those developed by Trivisonno and Hendricks (1977), and the thermophysical properties routines for several fluids (Hendricks and colleagues, 1973, 1975), or any of those supplied from the organizations list above, thermophysical properties may be accurately and quickly compared on such complex charts as T-S, H-S, H-P, F-T, G-T, etc., see Fig. 6. These charts, which are useful in cycle analysis, also serve to rapidly assess descriptions of the PVT surface and its derivatives which, in turn, can establish corresponding states reduction parameters as required. The recent work of Mollerup extends the input parameter options of his mixtures program to (P, ϕ, x_α) where $\phi = T, V, H, S$ and (H, S, x_α) .

D. EXAMPLES OF THERMOPHYSICAL PROPERTIES RELATED TO DYNAMIC PROCESSES

D1. Thermophysical Properties

D1.1 A classic example of a thermodynamic property related to a dynamic process is sonic velocity. The dynamic wave motion is described completely and entirely by curvature of the PVT surface representing the fluid through which the wave is passing.

$$c_s^2 = \left(\frac{\partial P}{\partial \rho}\right)_s \quad (\text{D.1})$$

Thus sonic velocity is fixed by state and not process. This concept is modified somewhat at high frequencies.

⁵NBS - Gaithersburg, MD and Boulder, CO; IUPAC - Imperial College, London; HTFS - AERE Harwell, England; NASA - COMSIC U. of Georgia, Athens, GA; AGA - Arlington, VA; API - Washington, D.C.

D1.2 Joule-Thomson coefficients are also thermodynamic properties which are related to an expansion process. For an isenthalpic expansion,

$$\mu_o = \left(\frac{\partial T}{\partial P} \right)_H = \frac{1}{\rho C_p} \left[\frac{T}{\rho} \frac{\left(\frac{\partial P}{\partial T} \right)_\rho}{\left(\frac{\partial P}{\partial \rho} \right)_T} - 1 \right] \quad (D.2)$$

D1.3 Hendricks, Baron, and Peller (1972) demonstrated that for engineering purposes the reduced Joule-Thomson coefficient, see Fig. 7, was useful in grouping results for various simple fluids

$$\mu_{oR} = \frac{\mu_o}{\mu_{o,c}} \quad (D.3)$$

where

$$\mu_{o,c} = \left[\left(\frac{\partial P}{\partial T} \right)_\rho \right]^{-1}_{(PVT)_c} \sim \frac{T_c}{6P_c} \quad (D.4)$$

Figure 7 illustrates a reduced Joule-Thompson coefficient plot.

D1.4 Leinhard and Karimi (1977) and Alamgir applied the corresponding states principle to van der Waals spinodal locus (Leinhard and Karimi, 1977) and surface tension (Leinhard and Alamgir) for several fluids, see Fig. 8.⁶

D1.5 Simoneau and Hendricks (1977) used the principle to extend the calculations for equilibrium fluid properties into the metastable regions to the equivalent van der Waals minimum pressure locus or vapor spinodal, for use in two phase choked flow computations, see Fig. 9.

D1.6 Parameters such as $(\partial \ln \rho / \partial \phi_j) = \beta_j$ where $\phi_j = T, H, E, A$, or can be reduced to a single plot of sufficient accuracy for most heat transfer calculations. This has been demonstrated by Hendricks (1979). Figure 10 represents a reduced volumetric expansion factor plot $\beta_R = \beta_1 / (Z_c T_c)^{-1}$. These few examples serve to illustrate some of the principles of similarity.

D2. Heat and Mass Transfer Processes Related through the Principle

D.2.1 Choked Flows

Hendricks (1973, 1974) has shown that two-phase choked flow rates of several simple fluids can be grouped using the reduced mass flux (see Fig. 11(a))

$$W_R = \frac{W}{W^* \psi_a} \quad (D.5)$$

where

⁶It should follow that from Eq. (A.4)-footnote the extended corresponding states relation is:

$$\sigma_\alpha(\rho, T) = \sigma_o(\rho h, T/f) (f/h^{2/3})$$

$$W^* = \sqrt{\frac{P_c \rho_c}{Z_c}} \quad \psi_a = \begin{cases} 1 & \text{Classical Fluids} \\ f(T_R) & \text{Quantum Fluids} \end{cases} \quad (D.6)$$

also in good agreement are the reduced pressure profiles of Fig. 11(b) where the ordinate represents the ratio of the nozzle throat pressure to that of an isentropic gas expansion.

The parameter W^* ⁷ has been useful in grouping and predicting mass flux for several geometries in addition to multiple fluids, see Simoneau and Hendricks (1979).

Hendricks (1979a, 1979b) and Poolos (1979) have used this normalization procedure to group data for free jet flows through tubes with Borda and sharp edge inlets. For (Hendricks and Poolos, 1979) the tube was short, $L/D = 2$, but four inlet geometries were used; circular, square, rectangular and triangular.

More recently, Hendricks (1979c) has shown that for gases flowing through a simulated shaft seal configuration, a modified form

$$\frac{W_R \sqrt{T_{Ro}}}{P_{Ro}} = c \quad (D.7)$$

reduces to a universal curve for simple gases; for this simulated shaft seal geometry, c is nearly constant at 0.16. For a nozzle, $c = 1/5$.

In convergent tapered passages, the mass flux rate is reduced using the area \bar{A}_1

$$\bar{A}_1 = (2A_{exit} + A_{inlet})/3 \quad (D.8)$$

$$W = \dot{m}/A_1 \quad (D.9)$$

With this modification, the above relations are valid for convergent tapered passages as well.

D2.2 Heat Transfer

Hendricks (1977) has shown that there is good evidence that the transposed critical temperature, T_1^* ⁸ used in near critical heat transfer problems, can be related through the corresponding states principles.

⁷A more general form is:

$$W_\alpha^* = W_o^* \sqrt{f/h}$$

and for the ideal gas case,

$$\psi_a^2 = \text{Const.} \cdot \frac{k}{R} \left(\frac{2}{k+1} \right)^{(k+1/k-1)}$$

where k is the isentropic expansion coefficient.

T_1^* ⁸ represents the loci of maximum specific heats.

The transposed critical temperature locus T_1^* has been used by several investigators to classify heat transfer data where behavior was more gas like $T > T_1^*$ or more liquid like $T < T_1^*$ and processes which crossed the T_1^* locus could possess either a maximum or a minimum heat transport. This phenomenon is more fully discussed by Hendricks and colleagues (1962, 1966, 1970). The point is, that T_1^* appears to be a corresponding states parameter, some heat transfer data can be classified using this criteria and consequently the grouping of heat transfer and hydrodynamic data in this manner could also follow a similarity principle.

Two component models are commonly used to represent heat transfer and hydrodynamic data where distinct phase transitions occur (e.g., liquid-vapor). However, two component models are also used to represent HeI and HeII transitions below the temperature (Lee and colleagues, 1978), and in grouping data of classical fluids in the subcritical and supercritical pressure states (Hendricks and colleagues, 1962, 1970). Similarities in wall temperature profiles and flow stability between the subcritical and supercritical pressure heat transfer data for forced convection and the wall temperature-heat flux profiles and stability in "pool boiling" have been noted by several investigators (Graham, Hendricks, and Ehlers, 1964; Hendricks and colleagues, 1962; Sabersky and Hauptmann, 1967).

D2.2.2 Subcritical Convection

Hendricks and Papell (1977) used the principle to group forced convection subcooled nucleate boiling data for neon, nitrogen and water. The reduced temperature difference was expressed as

$$\Delta T_R = \frac{(z_c - \frac{\omega}{4})}{3} q_R^{1/4} \exp \left[- \frac{4P_R}{7(z_c - \frac{\omega}{4})} \right] \quad (D.10)$$

where

$$q_R = q/q_R^* \quad (D.11)$$

$$\omega = -1 - \log P_v \Big|_{T_R=0.7} \quad (D.12)$$

and

$$q_k^* = 0.16 h_{fg} (g \sigma \rho_g^2 \rho_{fg})^{1/4} \quad (D.13)$$

is the Kutateladze expression for critical heat flux

D2.2.3 Volumetric Expansion

Yaskin and colleagues (1977) used the volumetric expansion parameter in the form of $(1 + \beta_1 \Delta T)$ in successfully grouping heat transfer data for fluid helium including the near critical region. A similar parameter $[1 + |(\partial \ln \rho)/(\partial H)|_P \Delta H]$ was used by Hendricks (1977), to group fluid nitrogen and fluid oxygen data. However it did not group the data as well as the work of Yaskin and colleagues (1977). Here $\Delta T = T_w - T_B$ and $\Delta H = H_w - H_B$.

Hendricks (1977) showed that heat transfer data for fluids nitrogen and oxygen were qualitatively related using the principle; however, assessment of the effects of buoyancy, relative velocity parameters, etc. which also grouped these data, must be made before the simulation can be made quantitative.

It was shown by Simoneau and Hendricks (1964), that for gases, the heat transfer coefficient can be written

$$h_c = CG^{0.8} d^{-0.2} \sqrt{T_B/T_W} \quad (D.14)$$

and it can be shown that for an ideal gas

$$[1 + \beta_1(T_W - T_B)]^{1/2} = \sqrt{T_B/T_W} \quad (D.15)$$

(Note: C related to M .) This work was reviewed (Hendricks, 1979d), and many sets of heat transfer data can be grouped using volumetric expansion parameters φ_k and the reduced Nusselt number Nu_R , see Fig. 12:

$$Nu_R = \frac{Nu_{exp}}{Nu_{calc}} \sim \psi_k^{-m} \quad m \approx \frac{1}{2} \quad (D.16)$$

where

$$Nu_{calc} = 0.023 Re^{0.8} Pr^{0.4} \quad (D.17)$$

$$\psi_k = 1 + \beta_k \Delta \varphi_k = 1 + \beta_k (\varphi_W - \varphi_B)_k \quad \text{no sum on } k \quad (D.18)$$

$$\beta_k = - \left(\frac{\partial \ln \rho}{\partial \varphi_k} \right)_p \quad (D.19)$$

and

$$\left. \begin{array}{lll} \varphi_1 = T & \varphi_3 = F & \varphi_5 = E \\ \varphi_2 = H & \varphi_4 = G & \varphi_6 = \mu_1 \end{array} \right\} \quad (D.20)$$

Using the Gibbs energy parameter φ_4 , the data group well indicating that if the process were constructed using Gibbs potential, a better understanding would ensue. Recall that the scaled equation of state in the critical region is based on chemical potential which reduces to Gibbs potential for pure fluids.

The coefficient, β_k , has been shown (D1.6), to obey the principle of corresponding states; for $k = 1$ the normalizing parameter $(Z_c T_c)^{-1}$, as noted before, is suitable for engineering calculations.

D2.2.4 Volumetric Expansion and Density Fluctuations

Hendricks and colleagues (1979) have demonstrated that the parameter β_j is related to density fluctuations and in turn to turbulent flows with substantial temperature and or pressure gradients. Further, the fluctuation components are related through the equations of state where the density can be written

$$-\frac{\rho'}{\bar{p}} = \beta_2 H' + k_T P' + \rho_1 x_1' \quad (D.21)$$

where the "primed" terms represent the fluctuating terms. Introducing fluctuation density ($\rho = \bar{\rho} + \rho'$), pressure ($P = \bar{P} + P'$), enthalpy ($H = \bar{H} + H'$), and velocity ($u = \bar{u} + u'$) into the conservation equations cited in the INTRODUCTION, the turbulent momentum and energy transport, for developed pipe flow, were written (Hendricks and colleagues, 1979),

$$-\tau_t = \bar{\rho} \overline{u'v'} + \bar{v} \overline{\rho'u'} + \overline{\rho'u'v'} \quad (D.22)$$

$$-q_t = \bar{\rho} \overline{v'H'} + \bar{v} \overline{\rho'H'} + \overline{\rho'v'H'} \quad (D.23)$$

At very small temperature differences, the convective heat transfer is enhanced and it follows that the Rayleigh number would be an important parameter

$$g \rho c_p \beta \Delta T d^3 / \nu_a \lambda$$

See for example (Hendricks, Simoneau, and Smith, 1970) and the careful work of Irie and colleagues (1978), which would follow more closely the scaling laws; compare also the work of Lee and colleagues (1978) and Klipping.

E. POSSIBLE GENERALIZATIONS

E1. Background

The usefulness of the similarity principles in grouping results for processes with heat and momentum transport has been demonstrated in the previous sections. The fact that the results obtained with cryogenic fluids can also be applied to a widely different fluid such as water is extremely useful.

Following the extended corresponding states principles of sections A and B, in this section we propose to "scale" fluids as

$$P_o = Ph/f; \eta_o = \eta F_\eta; T_o = T/f; \lambda_o = \lambda F_\lambda; V_o = Vh$$

and transform thermophysical properties as

$$\varphi^T = H(h, f, \phi, \theta, F_\eta, F_\lambda \dots) \varphi$$

such that solutions of the fundamental equations of motion could be transformed independent of fluid characteristics.⁹

⁹To assign a characteristic length common to both molecular and continuum mechanics, it will be necessary to separate the effects of h , F_λ , F_η (Eqs. (A.4)-(A.16-17)). Then perhaps one could assign

$$L_\alpha = L_o \left(\xi_\alpha^c / \xi_o^c \right) \psi_g$$

however the assessment of L_α is difficult as transport property measurements themselves represent solutions of the conservation equations and the character of ψ_g must be such as to give L_α independent of cluster size where appropriate; its definition is beyond the scope of this paper.

E2. Calculation of Critical Mass Flux Using the Generalized Transform

The thermophysical properties program of Hendricks and colleagues (1975), which was based on the work of Bender (1970), was used to both compute the properties of the reference fluid and the necessary constants for the transform. The analytical program used by Simoneau and Hendricks (1979) was adapted to calculate the flow rates and pressure ratios for two phase choked flow through a nozzle.¹⁰ The pressure and temperature of the working fluid were converted to the pressure and temperature of the reference fluid; computations then proceeded as usual for a single component reference fluid.

The conversion of pressure and temperature follows

$$P_{R_\alpha} = P_{R_0} \quad T_{R_\alpha} = T_{R_0} \quad (E.3)$$

$$P_\alpha = P_0 \frac{Z_\alpha^c}{Z_0^c} \frac{f}{h} \quad T_\alpha = f T_0 \quad (E.4)$$

where

$$h = \frac{M_0}{M_\alpha} \frac{\rho_0^c}{\rho_\alpha^c} \Phi \quad f = \frac{T_\alpha^c}{T_0^c} \Theta \quad (E.5)$$

$$\Phi = 1 + (\omega_\alpha - \omega_0) [b_1(V_R - b_2) - b_3(V_R - b_4) \ln T_R] \quad (E.6)$$

$$\Theta = 1 + (\omega_\alpha - \omega_0) [a_1 - a_2 \ln T_R + (a_3 - a_4/T_R)(V_R - a_5)] \quad (E.7)$$

The values of Φ and Θ were adopted from McCarty (1977).

Some calculated reduced critical mass flux values are given in Table 4 for oxygen or nitrogen flowing through a nozzle. The computed values of reduced mass flux are in good agreement, as can be seen in Tables 5 and 6 which were constructed using the various reference fluids methane, nitrogen, oxygen. The calculated values are compared to the experimental data, Fig. 12, of Hendricks, Simoneau, and Barrows (1976) in Figs. 13 to 19. In general the nitrogen and oxygen data points are in excellent agreement, independent of the working fluid, providing they are normalized (Hendricks, Simoneau, and Barrows, 1976), see Fig. 12. The theoretical calculations are for the most part, in good agreement independent of the working fluid and the reference fluid used to calculate the results. The agreement between oxygen data and theory using oxygen properties is in general quite good for this complex flow and the same comment can be made for nitrogen.

These results lend credence to the similarity principles delineated earlier and in part substantiate their validity. The work for the thermophysical properties people is extensive as application of the similarity principles requires accurate data bases for classes of fluids. In fluid mechanics and heat transfer accurate definitive experiments will be required.

¹⁰The homogeneous equilibrium fluid model of the nozzle with no slip between phases was chosen because of its relative simplicity and agreement with experimental results.

F. SUMMARY

The principle of similarity as applied to thermophysical properties with emphasis on the critical region and fluid mechanics, has been reviewed and shown, qualitatively, to group the experimental results of several physical processes. Using these principles, it is proposed that solutions can be developed which are independent of BOTH the fluid characteristics and the physical nature of the coordinates. To illustrate this proposal, two phase choked flow of fluid α was computed using fluid β where α and β were either methane, nitrogen or oxygen; the results are in good agreement with extensive oxygen and nitrogen data.

SYMBOLS

A_1	area
$a, b, a_1, b_1, \Omega_a, \Omega_b, m$	constants
B	scaling parameter, Eq. (B.6)
C_p	specific heat, constant pressure
C_v	specific heat, constant volume
c	constant
c_s	speed of sound
c_T, c_ρ	coefficients of temperature and density difference terms, Eq. (B.20)
D	diffusion coefficient
D, E	constants, Eqs. (B.9), (B.10), and (B.12)
d	diameter
F	free energy, Helmholtz
F_λ	scale function for thermal conductivity
F_η	scale function for viscosity
f	scale function, fluid α to reference fluid
f_f	friction factor
f_1	bounding function, Eq. (B.19)
G_m	configurational Gibbs free energy; mixture
G_o	configurational Gibbs free energy, reference fluid
G_α	configurational Gibbs free energy, fluid α
G	Gibbs free energy
$G_{\alpha,0}^{[]}$	scale parameter, Eqs. (A.30), (A.27), and (A.28)

g	body force
H	static enthalpy
H_0	total enthalpy
h	scale function, fluid α to reference fluid
h_t	heat transfer coefficient
$h(y)$	scaling function, Eq. (B.8)
k	isentropic expansion coefficient
k_B	Boltzman's constant
k_T	isothermal compressibility
M	molecular weight
\dot{m}	mass flow rate
Pr	Prandtl number
P	pressure
$Q_{\alpha,0}$	scale parameter, Eq. (A.29)
q	heat flux
q_k^*	reference heat flux, Kutateladze, Eq. (D.13)
q_1	fluid scale parameter, Eq. (B.22)
R	gas constant
S	entropy
T	temperature
T_1^*	transposed critical temperature (pseudo critical temperature)
t	time
t_0	reference time
u, v	velocity components
V	specific volume
$W = \rho u$	mass flux
$W^* = \sqrt{P_c \rho_c / Z_c}$	flow normalizing parameter
X	coordinate scale
x	scaling variable, Eq. (B.5)

x_α	mole fraction
y	scaling parameter, Eq. (B.7)
$Z = PV/RT$	compressibility

GREEK

A, A'	amplitude of power law relation for specific heat
$\alpha, \beta, \gamma, \delta$	scaling law parameters, Eqs. (B.4) and B.12)
β_j	volumetric or energy expansion coefficient
Γ	scaling parameter, Eq. (B.11b)
Γ'	scaling parameter, Eq. (B.11c)
η	viscosity
θ	shape factor
κ	Enskog parameter, Eqs. (A.33) and (A.34)
Λ	numerical level constant, Eq. (B.19)
λ	thermal conductivity
λ_I	HeI - HeII transition locus
μ	chemical potential
μ_0	Joule-Thomson coefficient
ν	critical exponent, Eq. (B.14)
ν_a	dynamic viscosity
ξ	correlation length, Eq. (B.14)
ξ_1	viscosity normalization parameter, Eq. (A.5)
ξ_2	thermal conductivity normalization parameter, Eq. (A.7)
ξ_3	diffusion normalization parameter, Eq. (A.9)
ξ_4	surface tension normalization parameter, Eq. (A.11)
$\xi_{\alpha, \beta}$	binary interaction parameter
ρ	density
σ_s	surface tension
τ	shear force
Φ	shape factor

ϕ	property parameter, Eqs. (D.19) and (E.2)
$\phi_{\alpha,\beta}$	binary interaction parameter
ϕ_1	constant, Eq. (B.23)
χ	scale parameter, Eqs. (A.27), (A.28), and (A.33)
χ_T	symmetrized isothermal compressibility
ψ	correction parameter, Eq. (D.6)
ψ_k	volumetric or energy expansion parameter, Eq. (D.17)
$\omega = -1 - \log_{10} P_R \Big _{T_R} = 0.7$	acentric factor, Eq. (D.12)

Subscripts:

B	bulk
CIC	critical isochore
CIT	critical isotherm
CXC	coexistence curve
c	critical
m	mixture
o	reference fluid
R	reduced
T	throat of nozzle
w	wall
0,0	"zero-density" reference state
α	working fluid
α,β	binary interaction
$\alpha\alpha,\beta\beta$	pure component interactions

Superscripts:

c	critical
—	average
~	background or "normal conditions"
'	fluctuating
*	normalized

REFERENCES

- Abe, Y., and J. Kestin (1979). Viscosity of Natural Gases. Rev. Gen. Therm., 18, 11-15.
- Alamgir, Md., and J. H. Lienhard. The Temperature Dependence of Surface Tension of Pure Fluids. To be published.
- Baker, G. A., B. G. Nickel, and D. I. Meiron (1978). Critical Indices from Perturbation Analysis of the Callan-Symanzik Equation. Phys. Rev., B17, 1365-1374.
- Bar, R. S., and J. V. Sengers (1977). Thermal Conductivity of Steam in the Critical Region. In A. Cezairliyan, (Ed.), Symposium on Thermophysical Properties, 7th American Society of Mechanical Engineers, New York, pp. 822-830.
- Basu, R. S., and J. V. Sengers (1979). Viscosity of Nitrogen Near the Critical Point. J. Heat Transfer, 101, 3-8.
- Basu, R. S., J. V. Sengers, and J. T. R. Watson, In 9th International Conference on the Properties of Steam. (to be published).
- Bender, E. (1970). Equation of State Exactly Representing the Phase Behavior of Pure Substances. Proceedings of the Fifth Symposium on Thermophysical Properties. American Society of Mechanical Engineers, New York, pp. 227-235.
- Bird, R. B., W. E. Stewart, and E. N. Lightfoot (1960). Transport Phenomena, Wiley, New York.
- Bittker, D. A., and V. J. Scullin (1972). General Chemical Kinetics Computer Program for Static and Flow Reactions, with Application to Combustion and Shock-Tube Kinetics. NASA TN-D6568.
- Brock, J. R., and R. B. Bird (1955). Surface Tension and the Principle of Corresponding States. AIChE J., 1, 174-177.
- Cabeci, T., and A. M. O. Smith (1974). Analysis of Turbulent Boundary Layers, Applied Mathematics and Mechanics, Vol. 15, Academic Press, New York.
- Dean, D. E., and L. I. Sriel (1965). The Viscosity of Nonpolar Gas Mixtures at Moderate and High Pressures. AIChE J., 11, 526-532.
- Fisher, M. E. (1967). The Theory of Equilibrium Critical Phenomena. Rep. Progr. Phys., G. B., 30, 615-730.
- Goldmann, K. (1963). Heat Transfer to Supercritical Water at 5000 psi Flowing at High Mass Flow Rates Through Round Tubes. In International Developments in Heat Transfer, American Society of Mechanical Engineers, pp. 561-568.
- Gordon, S., and B. J. McBride (1976). Computer Programs for Calculation of Complex Chemical Equilibrium Compositions, Rocket Performance, Incident and Reflected Shocks, and Chapman-Jouquet Detonations, NASA SP-273 Rev.
- Graham, R. W., R. C. Hendricks, and R. C. Ehlers (1965). Analytical and Experimental Study of Pool Heating of Liquid Hydrogen over a Range of Accelerations. NASA TN D 1883.
- Hanley, H. J. M. (1977). Prediction of the Thermal Conductivity of Fluid Mixtures. Proceedings of the Seventh Symposium on Thermophysical Properties, American Society of Mechanical Engineers, New York, pp. 668-673.
- Hanley, H. J. M., K. E. Gubbins, and S. Murad (1977). A Correlation of the Existing Viscosity and Thermal Conductivity Data of Gases and Liquid Ethane. J. Phys. & Chem. Ref. Data, 6, 1167-1180.
- Hanley, H. J. M., W. M. Haynes, and R. D. McCarty (1977). The Viscosity and Thermal Conductivity Coefficients for Dense Gaseous and Liquid Methane. J. Phys. & Chem. Ref. Data, 6, 597-609.
- Hanley, H. J. M., R. D. McCarty, and W. M. Haynes (1974). The Viscosity and Thermal Conductivity Coefficients for Dense Gaseous and Liquid Argon, Krypton, Xenon, Nitrogen, and Oxygen. J. Phys. & Chem. Ref. Data, 3, 979-1018.
- Hanley, H. J. M., R. D. McCarty, and J. V. Sengers (1974). Viscosity and Thermal Conductivity Coefficients of Gaseous and Liquid Oxygen. NASA CR-2440.
- Hanley, H. J. M., J. V. Sengers, and J. F. Ely (1976). On Estimating Thermal Conductivity Coefficients in the Critical Region of Gases. In P. G. Klemens and T.-K. Chu, (Eds.), Thermal Conductivity - 14, Plenum Press, New York, pp. 383-407.

- Hansen, A. G. (1964). Similarity Analyses of Boundary Value Problems in Engineering, Prentice-Hall, Englewood Cliffs, New Jersey.
- Hendricks, R. C. (1974). Normalizing Parameters for the Critical Flow Rate of Simple Fluids Through Nozzles. In K. Mendelssohn, (Ed.), International Cryogenic Engineering Conference, 5th., IPC Science and Technology Press, pp. 278-281.
- Hendricks, R. C. (1977). Simulation of Heat Transfer Characteristics of LOX. NASA TM X-73634. 17th AIChE/ASME Nat. Heat Trans. Conf., Salt Lake City, Utah.
- Hendricks, R. C. (1979a). A Reduced Volumetric Expansion Factor Plot. Presented at the Cryogenic Engineering Conference/International Cryogenic Materials Conference, University of Wisconsin, Madison, Wisconsin, Paper CD 6.
- Hendricks, R. C. (1979b). Some Aspects of a Free Jet Phenomena to 105 L/D in a Constant Area Duct. To be presented at the International Congress of Refrigeration, 15th, Venice, Italy, Paper B1-78.
- Hendricks, R. C. (1979c). A Free Jet Phenomena in a 90°-Sharp Edge Inlet Geometry. Presented at the Cryogenic Engineering Conference/International Cryogenic Materials Conference, University of Wisconsin, Madison, Wisconsin, Paper CC7.
- Hendricks, R. C. (1979d). Some Flow Characteristics of Conventional and Tapered High Pressure Drop Simulated Seals. To be presented at the American Society of Lubrication Engineers/American Society of Mechanical Engineers Joint Lubrication Conference, Dayton, Ohio, Preprint 79-LC-3B-2.
- Hendricks, R. C. (1979e). Volume-Energy Parameters and Turbulent Flow Density Fluctuations, NASA TP-
- Hendricks, R. C., A. K. Baron, and I. C. Peller (1975). GASP - A Computer Code for Calculating the Thermodynamic and Transport Properties for Ten Fluids: Parahydrogen, Helium, Neon, Methane, Nitrogen, Carbon Monoxide, Oxygen, Fluorine, Argon, and Carbon Dioxide. NASA TN D-7808.
- Hendricks, R. C., R. W. Graham, Y. Y. Hsu, and R. Friedman (1966). Experimental Heat-Transfer Results for Cryogenic Hydrogen Flowing in Tubes at Subcritical and Supercritical Pressures to 800 Pounds per Square Inch Absolute. NASA TN D-3095.
- Hendricks, R. C., R. W. Graham, Y. Y. Hsu, and A. A. Medeiros (1962). Correlation of Hydrogen Heat Transfer in Boiling and Supercritical Pressure States. Am. Rocket Soc. J., 32, 244-252.
- Hendricks, R. C., R. B. McClintock, and G. J. Silvestri (1977). Revised International Representations for the Viscosity of Water and Steam and New Representations for the Surface Tension of Water. J. Eng. Power, 99, 664-678.
- Hendricks, R. C., and S. S. Papell (1978). Estimating Surface Temperature in Forced Convection Nucleate Boiling - A Simplified Method. In K. D. Timmerhaus, (Ed.), Advances in Cryogenic Engineering, Vol. 23, Plenum Press, New York, pp. 301-304.
- Hendricks, R. C., I. C. Peller, and A. K. Baron (1972). Joule-Thomson Inversion Curves and Related Coefficients for Several Simple Fluids. NASA TN D-6807.
- Hendricks, R. C., I. C. Peller, and A. K. Baron (1973). WASP-A Flexible Fortran IV Computer Code for Calculating Water and Steam Properties. NASA TN D-7391.
- Hendricks, R. C., and N. Poolos (1979f). Critical Mass Flux Through Short Borda Type Inlets of Various Cross-Sections. To Be presented at the International Congress of Refrigeration, 15th, Venice, Italy, Paper B1-77.
- Hendricks, R. C., and R. J. Simoneau (1973). Application of the Principle of Corresponding States to Two-Phase Choked Flows. NASA TM X-68193.
- Hendricks, R. C., R. J. Simoneau, and R. F. Barrows (1976). Two - Phase Choked Flow of Subcooled Oxygen and Nitrogen. NASA TN D-8169.
- Hendricks, R. C., R. J. Simoneau, and R. J. Smith (1970). Survey of Heat Transfer to Near Critical Fluids. NASA TN D-5886.
- Hendricks, R. C., V. M. Yeroshenko, L. A. Yaskin, and D. D. Starostin (1979g). Bulk Expansion Factors and Density Fluctuations in Heat and Mass Transfer. To be presented at the International Congress of Refrigeration, 15th, Venice, Italy. Paper B1-19.
- Hinze, J. O. (1975). Turbulence, 2nd ed. McGraw-Hill, New York.
- Hohenberg, P. C., and B. I. Halperin (1977). Theory of Dynamic Critical Phenomena. Rev. Mod. Phys., 49, 435-479.
- Irie, F., T. Matshushita, M. Takeo, G. Clipping, K. Luders, and U. Ruppert (1978). Heat Transfer to Helium in the Near Critical Region. In K. D. Timmerhaus, (Ed.), Advances in Cryogenics, Vol. 23, Plenum Press, New York, pp. 326-332.

- Jossi, J. A., L. I. Stiel, and G. Thodos (1962). The Viscosity of Pure Substances in the Dense Gaseous and Fluid Phases. AIChE J., 8, 59-63.
- Kawasaki, K. (1970). Kinetic Equations and Time Correlation Functions of Critical Fluctuations. Ann. Phys., 61, 1-56.
- Kestin, J. (1978). A Course in Thermodynamics, Vol I and II, Rev. ed. Hemisphere, Washington.
- Leach, J. W. (1967). Molecular Structure Correction for Applying the Theory of Corresponding States to Non-Spherical Pure Fluids and Mixtures. PhD Thesis, Rice University, Houston, Texas.
- Leach, J. W., P. S. Chappellear, and T. W. Leleand (1966). Properties of Hydrocarbon and Quantum Gas Mixtures from the Corresponding-States Principle. In Proceedings of the American Petroleum Institute, Vol. 46, pp. 223-234.
- Lee, J. Y., S. Caspi, S. C. Soloski, and T. H. K. Frederking (1978). Thermo-Osmotic Convection of Cryo-Fluid He II at Incipient Lamda Transitions Above the Critical Pressure. In International Heat Transfer Conference, 6th, Vol. 3. National Research Council of Canada, Toronto, Ontario, pp. 209-214.
- Le Guillou, J. C., and J. Zinn-Justin (1977). Critical Exponents for the n-Vector Model in Three Dimensions from Field Theory. Phys. Rev. Lett., 39, 95-98.
- Levelt Sengers, J. M.H. (1974). From Van Der Waal's Equation to the Scaling Laws. Physica, 73, 73-106.
- Levelt Sengers, J. M. H., W. L. Greer, and J. V. Sengers (1976). Scaled Equations of State Parameters for Gases in the Critical Region. J. Phys. & Chem. Ref. Data, 5, 1-51.
- Levelt Sengers, A., R. Hocken, and J. V. Sengers (1977). Critical-Point Universality and Fluids. Phys. Today, 30, 42-51.
- Levelt Sengers, J. M. H., and J. V. Sengers (1976). Universality of Critical Behavior in Gases. Phys. Rev., A12, 2622-2627.
- Lienhard, J. H., and A. Karim (1977). Corresponding States Correlations of the Vapor Spinodal Lines. ASME Paper 77-HT-20.
- McCarty, R. D. (1977). A Comparison of Mathematical Models for the Prediction of LNG Densities. NBS IR 77-867.
- Misic, D. and G. Thodos (1961). The Thermal Conductivity of Hydrocarbon Gases at Normal Pressures. AIChE J., 7, 264-267.
- Mollerup, J. (1975). Correlated and Predicted Thermodynamic Properties of LNG and Related Mixtures in the Normal and Critical Regimes. In K D. Timmerhaus, (Ed.), Advances in Cryogenic Engineering, Vol. 20, Plenum Press, New York, pp. 172-194.
- Mollerup, J. (1978a). Thermodynamic Properties of Natural Gas, Petroleum Gas and Related Mixtures: Enthalpy Predictions. In K. D. Timmerhaus, (Ed.), Advances in Cryogenic Engineering, Vol. 23, Plenum Press, New York, pp. 550-560.
- Mollerup, J. (1978b). Thermodynamic Properties from Corresponding States Theory. American Institute of Chemical Engineers 71st Annual Meeting, Miami Beach, Fla., Paper 45 G.
- Moore, F. K. (Ed.), (1964). Theory of Laminar Flows, Vol. IV, High Speed Aerodynamics and Jet Propulsion, Princeton University Press, Princeton, New Jersey.
- Nagashima, A. (1977). Viscosity of Water Substance - New International Formulation and its Background, J. Phys. & Chem. Ref. Data, 6, 1133-1166.
- Ohta, T. (1977). Multiplicative Renormalization of the Anomalous Shear Viscosity in Classical Liquids. J. Phys., C10, 791-793.
- Ohta, T., and K. Kawasaki (1976). Mode Coupling Theory of Dynamic Critical Phenomena for Classical Liquids, I. Progr. Theor. Phys., 55, 1384-1395.
- Oxtoby, D. W., and W. M. Gelbart (1974). Shear Viscosity and Order Parameter Dynamics of Fluids Near the Critical Point, J. Chem. Phys., 61, 2957-2963.
- Perl, R. and R. A. Ferrell (1972). Decoupled-Mode Theory of Critical Viscosity and Diffusion in the Binary-Liquid Phase Transition. Phys. Rev., A6, 2358-2369.
- Rainwater, J. C., and H. J. M. Hanley (1977). Prediction of the Transport Properties of Natural Gas and Similar Mixtures. Advances in Cryogenic Engineering, Vol. 23, Plenum Press, New York, pp. 561-565.

- Sabersky, R. H., and E. G. Hauptmann (1967). Forced Convection Heat Transfer to Carbon Dioxide Near the Critical Point. Int. J. Heat & Mass Transf., 10, 1499-1508.
- Schaefer, C. A., and G. Thodos, (1959). Thermal Conductivity of Diatomic Gases: Liquid and Gaseous States. AIChE J., 8, 367-372.
- Scheffler, K., N. Rosener, J. Straub, and H. Grigull (1978). New International Standard for Dynamic Viscosity of Water and Steam. Brennst. Wärme-Kraft., 30, 73-78.
- Schlichting, H. (1968). Boundary Layer Theory, McGraw-Hill, New York.
- Sengers, J. V. (1972a). Transport Processes Near the Critical Point of Gases and Binary Liquids in the Hydrodynamic Regime. Ber. Bunsenges. Phys. Chem., 76, 234-249.
- Sengers, J. V. (1972b). Transport Properties of Gases and Binary Liquids Near the Critical Point. NASA CR-2112.
- Sengers, J. V. (1973). Transport Properties of Gases and Binary Liquids Near the Critical State. In J. Kestin, (Ed.), Transport Phenomena-1973, AIP Conference Proceedings No. 11, American Institute of Physics, New York, pp. 229-277.
- Sengers, J. V., and J. M. H. Levelt Sengers (1977). In C. A. Croxton, ed., Progress in Liquid Physics. Wiley, London, Chap. 4.
- Sengers, J. V., and J. M. H. Levelt Sengers (1977). Concepts and Methods for Describing Critical Phenomena in Fluids. NASA CR-149665.
- Simoneau, R. J., and R. C. Hendricks (1964). A Simple Equation for Correlating Turbulent Heat Transfer to a Gas. NASA TM X-52011.
- Simoneau, R. J., and R. C. Hendricks (1977). Generalized Charts for Computation of Two-Phase Choked Flow of Simple Cryogenic Liquids. Cryogenics, 17, 73-76.
- Simoneau, R. J., and R. C. Hendricks (1979). Two-Phase Choked Flow of Cryogenic Fluids in Converging-Diverging Nozzles. NASA TP 1484.
- Slattery, J. C., and R. B. Bird (1958). Calculation of the Diffusion Coefficient of Dilute Gases and of the Self-Diffusion Coefficient of Dense Gases. AIChE J., 4, 137-142.
- Stiel, L. I., and G. Thodos (1962a). The Prediction of the Transport Properties of Pure Gaseous and Liquid Substances. In Progress in International Research on Thermophysical Properties, American Society of Mechanical Engineers, New York, pp. 352-365.
- Stiel, L. I., and G. Thodos (1962b). The Viscosity of Polar Gases at Normal Pressures. AIChE J., 8, 229-232.
- Svehla, R. A. (1962). Estimated Viscosities and Thermal Conductivities of Gases at High Temperatures. NASA TR-R-132.
- Trivisonno, R., and R. C. Hendricks (1977). GASPLOT- A Computer Graphics Program that Draws a Variety of Thermophysical Property Charts. NASA TN D-8507.
- Vicentini-Missoni, M., J. M. H. Levelt Sengers, and M. S. Green (1969). Scaling Analysis of Thermodynamic Properties in the Critical Region of Fluids. Nat. Bur. Stand. J. Res., 73A, 563-583.
- Yaskin, L. A., M. C. Jones, V. M. Yeroshenko, P. J. Giarratano, and V. D. Arp (1977). A Correlation for Heat Transfer to Supercritical Helium in Turbulent Flow in Small Channels. Cryogenics, 17, 549-553.
- Zelevnik, F. J. (1976). Thermodynamics. J. Math. Phys., 17, 1579-1610.
- Zelevnik, F. J., and S. Gordon (1968). Calculation of Complex Chemical Equilibria Ind. & Eng. Chem., 60, 27-57.

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- Sabersky, R. H., and E. G. Hauptmann (1967). Forced Convection Heat Transfer to Carbon Dioxide Near the Critical Point. Int. J. Heat & Mass Transf., 10, 1499-1508.
- Schaefer, C. A., and G. Thodos, (1959). Thermal Conductivity of Diatomic Gases: Liquid and Gaseous States. AIChE J., 8, 367-372.
- Scheffler, K., N. Rosener, J. Straub, and H. Grigull (1978). New International Standard for Dynamic Viscosity of Water and Steam. Brennst. Wärme-Kraft., 30, 73-78.
- Schlichting, H. (1968). Boundary Layer Theory, McGraw-Hill, New York.
- Sengers, J. V. (1972a). Transport Processes Near the Critical Point of Gases and Binary Liquids in the Hydrodynamic Regime. Ber. Bunsenges. Phys. Chem., 76, 234-249.
- Sengers, J. V. (1972b). Transport Properties of Gases and Binary Liquids Near the Critical Point. NASA CR-2112.
- Sengers, J. V. (1973). Transport Properties of Gases and Binary Liquids Near the Critical State. In J. Kestin, (Ed.), Transport Phenomena-1973, AIP Conference Proceedings No. 11, American Institute of Physics, New York, pp. 229-277.
- Sengers, J. V., and J. M. H. Levelt Sengers (1977). In C. A. Croxton, ed., Progress in Liquid Physics. Wiley, London, Chap. 4.
- Sengers, J. V., and J. M. H. Levelt Sengers (1977). Concepts and Methods for Describing Critical Phenomena in Fluids. NASA CR-149665.
- Simoneau, R. J., and R. C. Hendricks (1964). A Simple Equation for Correlating Turbulent Heat Transfer to a Gas. NASA TM X-52011.
- Simoneau, R. J., and R. C. Hendricks (1977). Generalized Charts for Computation of Two-Phase Choked Flow of Simple Cryogenic Liquids. Cryogenics, 17, 73-76.
- Simoneau, R. J., and R. C. Hendricks (1979). Two-Phase Choked Flow of Cryogenic Fluids in Converging-Diverging Nozzles. NASA TP 1484.
- Slattery, J. C., and R. B. Bird (1958). Calculation of the Diffusion Coefficient of Dilute Gases and of the Self-Diffusion Coefficient of Dense Gases. AIChE J., 4, 137-142.
- Stiel, L. I., and G. Thodos (1962a). The Prediction of the Transport Properties of Pure Gaseous and Liquid Substances. In Progress in International Research on Thermophysical Properties, American Society of Mechanical Engineers, New York, pp. 352-365.
- Stiel, L. I., and G. Thodos (1962b). The Viscosity of Polar Gases at Normal Pressures. AIChE J., 8, 229-232.
- Svehla, R. A. (1962). Estimated Viscosities and Thermal Conductivities of Gases at High Temperatures. NASA TR-R-132.
- Trivisonno, R., and R. C. Hendricks (1977). GASPLIT- A Computer Graphics Program that Draws a Variety of Thermophysical Property Charts. NASA TN D-8507.
- Vicentini-Missoni, M., J. M. H. Levelt Sengers, and M. S. Green (1969). Scaling Analysis of Thermodynamic Properties in the Critical Region of Fluids. Nat. Bur. Stand. J. Res., 73A, 563-583.
- Yaskin, L. A., M. C. Jones, V. M. Yeroshenko, P. J. Giarratano, and V. D. Arp (1977). A Correlation for Heat Transfer to Supercritical Helium in Turbulent Flow in Small Channels. Cryogenics, 17, 549-553.
- Zelevnik, F. J. (1976). Thermodynamics. J. Math. Phys., 17, 1579-1610.
- Zelevnik, F. J., and S. Gordon (1968). Calculation of Complex Chemical Equilibria Ind. & Eng. Chem., 60, 27-57.

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TABLE 1 Critical Power Laws

Property	Path	Power law
C_V^*/T^*	CIC, $\Delta T^* \geq 0$	$A \Delta T^* ^{-\alpha}$
C_V^*/T^*	CIC, $\Delta T^* \leq 0$	$A' \Delta T^* ^{-\alpha}$
$\Delta\phi^*$	CXC	$B \Delta T^* ^\beta$
χ_T^*	CIC, $\Delta T^* \geq 0$	$\Gamma \Delta T^* ^{-\gamma}$
χ_T^*	CXC	$\Gamma' \Delta T^* ^{-\gamma}$
$\Delta\mu^*$	CIT	$\pm D \Delta\phi^* ^\delta$

TABLE 2 Scaled Expressions for Thermodynamic Functions†

Chemical potential

$$\mu^* = \mu^*(\rho_c^*, T^*) + \Delta\phi^* |\Delta\phi^*|^{\delta-1} Dh(y)$$

Compressibility

$$\chi_T^{*-1} = |\Delta\phi^*|^{\delta-1} D \left[\delta h(y) + \frac{1}{\beta} (1-y) \frac{dh(y)}{dy} \right]$$

Helmholtz free energy density

$$F^* = A_O^*(T^*) + \rho^* \mu^*(\rho_c^*, T^*) + |\Delta\phi^*|^{\delta+1} Da(y)$$

Entropy density

$$-S = \frac{dA_O^*(T^*)}{dT^*} + \rho^* \frac{d\mu^*(\rho_c^*, T^*)}{dT^*} + |\Delta\phi^*|^{(1-\alpha)/\beta} \frac{D}{x_O^2} \frac{da(y)}{dy}$$

Heat capacity

$$-\frac{C_V^*}{T^*} = \frac{d^2 A_O^*(T^*)}{dT^{*2}} + \rho^* \frac{d^2 \mu^*(\rho_c^*, T^*)}{dT^{*2}} + |\Delta\phi^*|^{-\alpha/\beta} \frac{D}{x_O^2} \frac{d^2 a(y)}{dy^2}$$

Pressure

$$P^* = -A_O^*(T^*) + D \left\{ \Delta\phi^* |\Delta\phi^*|^{\delta-1} h(y) + |\Delta\phi^*|^{\delta+1} [h(y) - a(y)] \right\}$$

† $A_O^*(T^*)$ and $\mu^*(\rho_c^*, T^*)$ are analytic functions of T^* ; $\rho_c^* = 1$.

TABLE 3 Critical Region Parameters for a Number of Fluids Assuming
Effective Universal Exponents

	P_c , MPa	ρ_c , kg/m ³	T_c , K	x_o	D	$\xi_o \times 10^{10}$, m
³ He	0.11678	41.45	3.3099	0.489	3.16	2.7
⁴ He	.22742	69.6	5.1895	.369	2.86	2.2
Ar	4.865	535.	150.725	.183	2.43	1.6
Kr	5.4931	908.	209.286	.183	2.43	1.7
Xe	5.8400	1110.	289.734	.183	2.43	1.9
p-H ₂	1.285	31.39	32.935	.260	2.50	1.9
H ₂	3.398	313.9	126.24	.164	2.32	1.6
O ₂	5.043	1136.2	154.580	.183	2.36	1.6
H ₂ O	22.06	322.2	647.13	.100	1.28	1.3
D ₂ O	21.66	357.	643.89	.100	1.28	1.3
CO ₂	7.3753	467.8	304.127	.141	2.16	1.6
NH ₃	11.303	235.	405.4	.109	1.47	1.4
SF ₆	3.7605	730.	318.687	.172	3.06	2.0
CH ₄	4.595	162.7	190.555	.164	2.17	1.7
C ₂ H ₄	5.0390	215.	282.344	.168	2.32	1.9
C ₂ H ₆	4.8718	206.5	305.33	.147	2.17	1.8
C ₃ H ₈	4.247	221.	369.82	.137	1.96	2.0

Notes: $\alpha = 0.100$, $\beta = 0.355$, $\gamma = 1.190$, $\delta = 4.352$, $\nu = 0.633$,
E = 0.287.

TABLE 4 Comparison of Reduced Critical Mass Flux using the General
Thermophysical Property Transform Data from Reference

T, K	P, MPa	Reference fluid			Working fluid		$W_{R_{calc}}$	$W_{R_{exp}}$
		CH ₄	N ₂	O ₂	N ₂	O ₂		
94.6	492.	x			x		1.302	1.22
			x		x		1.34	
				x	x		1.324	
109.2	492.	x			x		1.109	1.05
			x		x		1.15	
				x	x		1.133	
119.1	492.	x			x		.922	.883
			x		x		.953	
				x	x		.852 (sat)	
139.3	492.	x			x		.385	.383
			x		x		.407	
				x	x		.400	
114.9	7.01	x				x	1.288	1.22
			x			x	1.324	
				x		x	1.309	
134.1	7.01	x				x	1.083	1.02
			x			x	1.123	
				x		x	1.106	
145.8	7.01	x				x	.898	.855
			x			x	.930	
				x		x	.835 (sat)	
169.8	7.11	x				x	.375	.368
			x			x	.395	
				x		x	.389	
224.6	7.11	x				x	.245	.230
			x			x	.256	
				x		x	.251	

REPRODUCIBILITY OF THE ORIGINAL DATA IS DOUBT

TABLE 5 Calculated Reduced Critical Mass Flux using the Principle of Similarity and the Homogeneous Equilibrium Model, Reference 65

P_R	P, MPa O_2 N_2	$T_R = 0.75$ $T_{O_2} = 116.1; T_{N_2} = 94.7 K$				$T_R = 0.942$ $T_{O_2} = 145.8; T_{N_2} = 119 K$				$T_R = 1.025$ $T_{O_2} = 158.6; T_{N_2} = 129.5 K$			
		$WR_{N_2-N_2}$ $WR_{N_2-O_2}$	$WR_{CH_4-O_2}$ $WR_{CH_4-N_2}$	$WR_{O_2-N_2}$ $WR_{O_2-CH_4}$	$WR_{CH_4-N_2}$ $WR_{CH_4-O_2}$	$WR_{N_2-N_2}$ $WR_{N_2-O_2}$	$WR_{CH_4-O_2}$ $WR_{CH_4-N_2}$	$WR_{O_2-N_2}$ $WR_{O_2-CH_4}$	$WR_{CH_4-N_2}$ $WR_{CH_4-O_2}$	$WR_{N_2-N_2}$ $WR_{N_2-O_2}$	$WR_{O_2-N_2}$ $WR_{O_2-CH_4}$	$WR_{CH_4-N_2}$ $WR_{CH_4-O_2}$	$WR_{CH_4-N_2}$ $WR_{CH_4-O_2}$
2.	10.16	6.834	1.62	1.61	1.57	1.59	1.56	1.25	1.28	1.24	1.16 sat	1.23	1.01
1.6	8.133	5.467	1.42	1.43	1.39	1.40	1.38	1.05	1.07	1.03	.95 sat	1.02	.77
1.2	6.10	4.1	1.21	1.22	1.18	1.19	1.17	.77	.80	.77	.68 sat	.75	.40
1.0	5.083	3.417	1.09	1.09	1.06	1.07	1.05	.6	.63	.59	.50 sat	.57	.25
.8	4.066	2.734	.95	.96	.92	.93	.92	.36	.39	.35	.34	.34	.18
.6	3.05	2.05	.79	.80	.76	.77	.76	.14	.14	.14	.14	.14	.13
.4	2.033	1.367	.59	.60	.56	.56	.55	.09	.09	.09	.09	.09	.08
.2	1.017	.683	.26	.28	.20	.21	.18	.04	.04	.04	.04	.04	.04
2.8	14.32	9.568						1.61	1.62	1.58	1.51	1.57	.60
1.4	7.116	4.784						.3	.31	.29	.29	.17	
.72	3.66	2.46											
.735	2.511												
.25	1.27	.854	.35	.38	.33	.33	.32					.56	

1st index = reference fluid used to calculate values.
2nd index = working fluid flowing through nozzle.

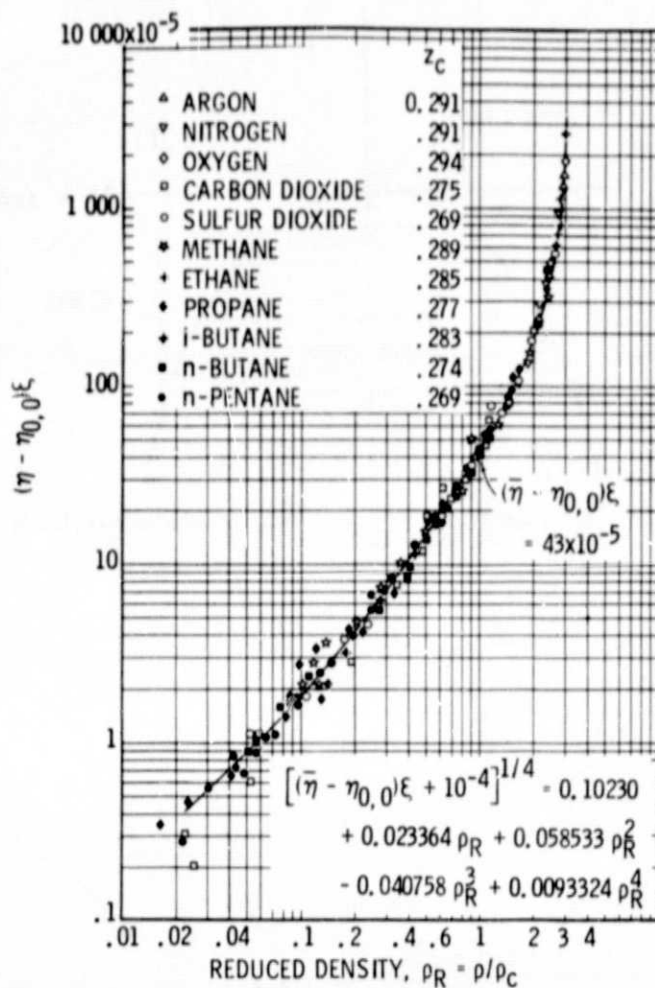


Fig. 1. Relationship between the residual viscosity modulus $(\bar{\eta} - \eta_{0,0})\xi$ and reduced density ρ_R for normally behaving substances, Jossi, Stiel, and Thodos (1962).

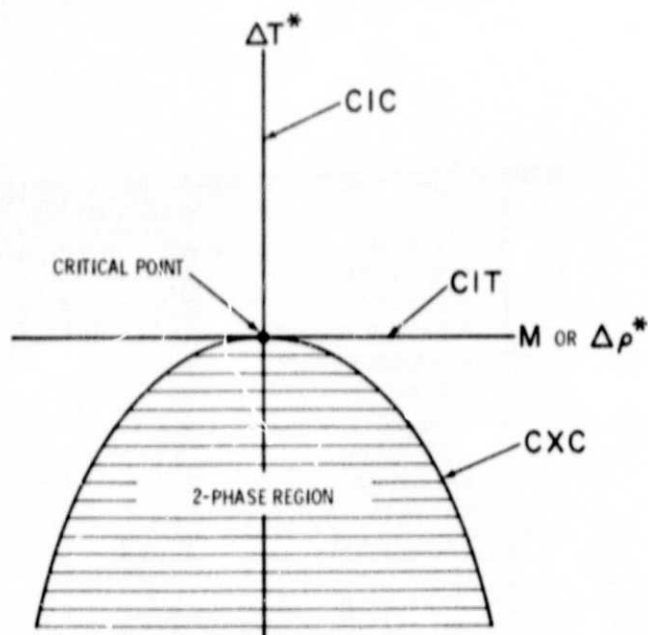


Fig. 2. Special paths in the ΔT^* versus $\Delta \rho^*$ plane for the definition of critical power laws.

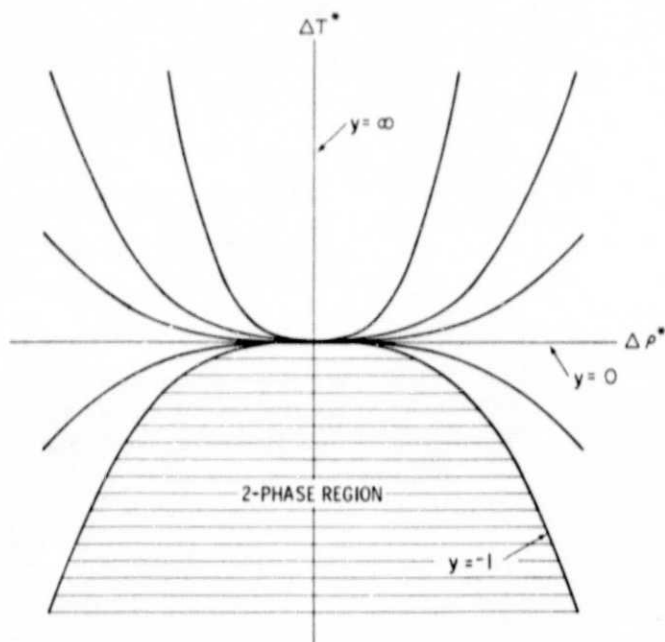


Fig. 3. Curves of constant y in the ΔT^* - $\Delta \rho^*$ plane.

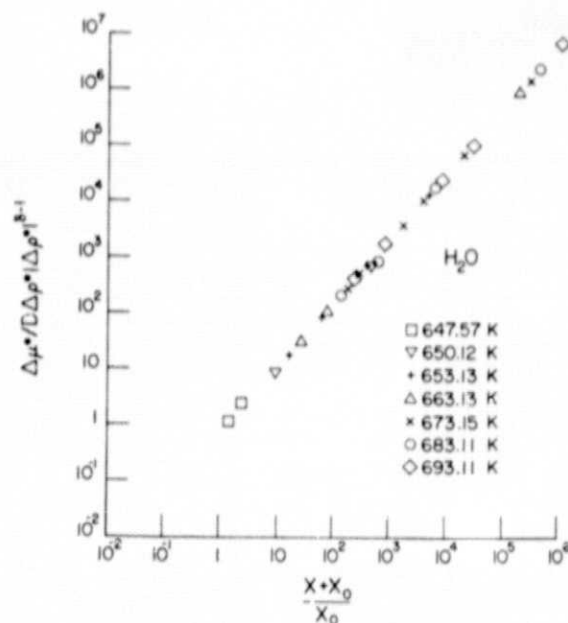


Fig. 4. Scaled chemical potential data for steam as a function of $y = (x + x_0)/x_0$. Reduction parameters are taken from Table III.

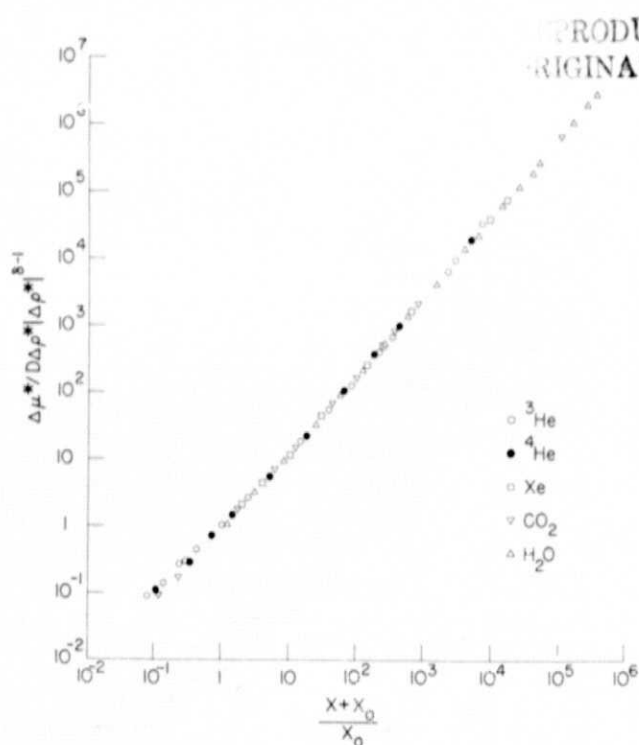
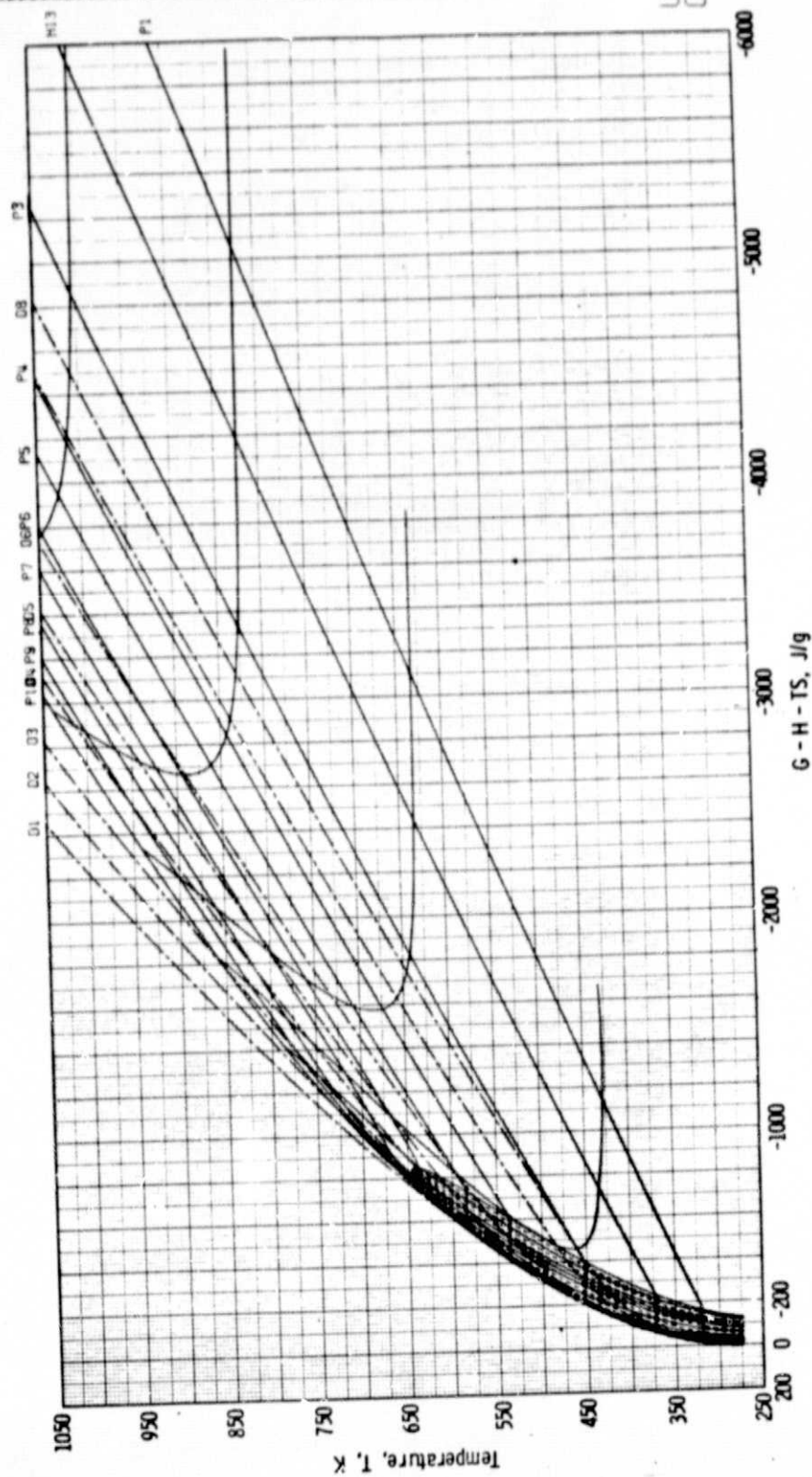
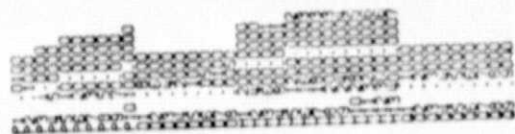


Fig. 5. Scaled chemical potential data for ^3He , ^4He , Xe, CO_2 , and H_2O as a function of $y = (x + x_0)/x_0$. Reduction parameters are taken from Table III.



UNITS=KJ=1
GAS=H2O

Fig. 6. Gibbs free energy chart for fluid water, Trivisonno and Hendricks (1977).

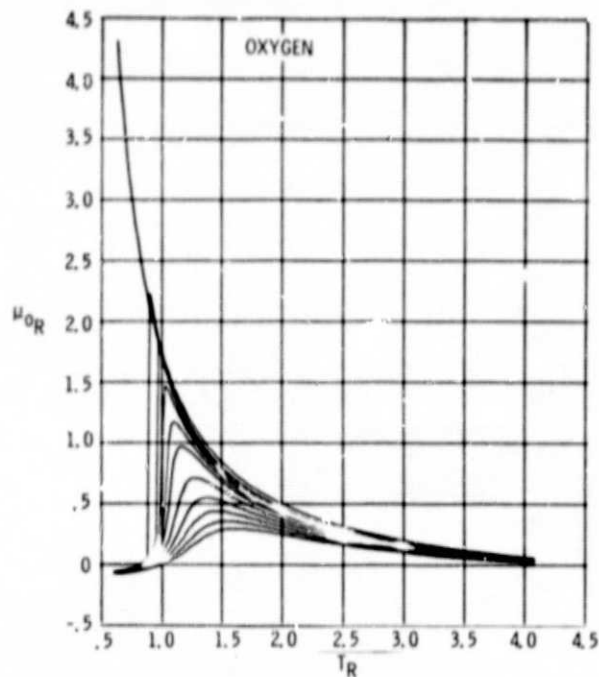


Fig. 7. Reduced Joule-Thomson coefficients as function of reduced temperature for selected isobars, Hendricks, Peller, and Baron (1972).

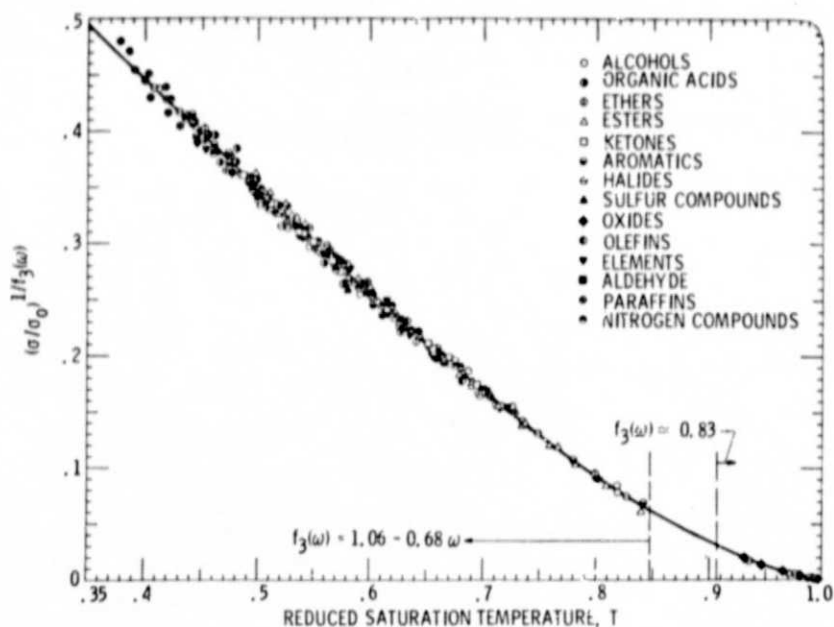


Fig. 8. Temperature dependence function for surface tension: low and moderate temperatures, Lienhard and Amir (1977).

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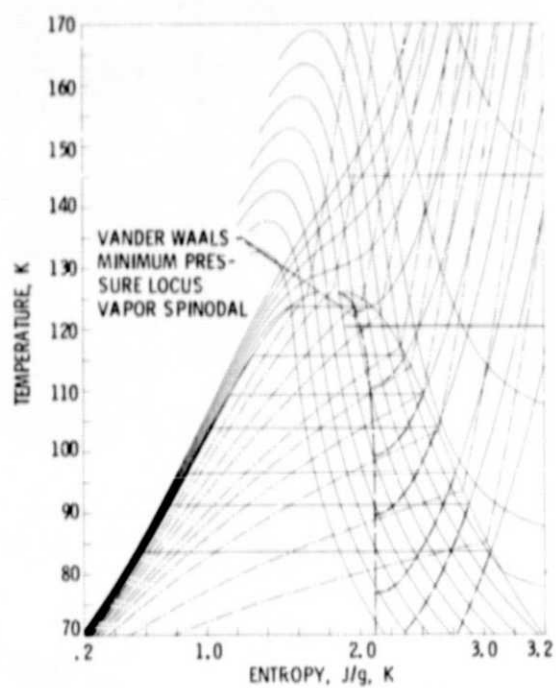


Fig. 9. Temperature-entropy chart with metastable locus, Simoneau and Hendricks (1977).

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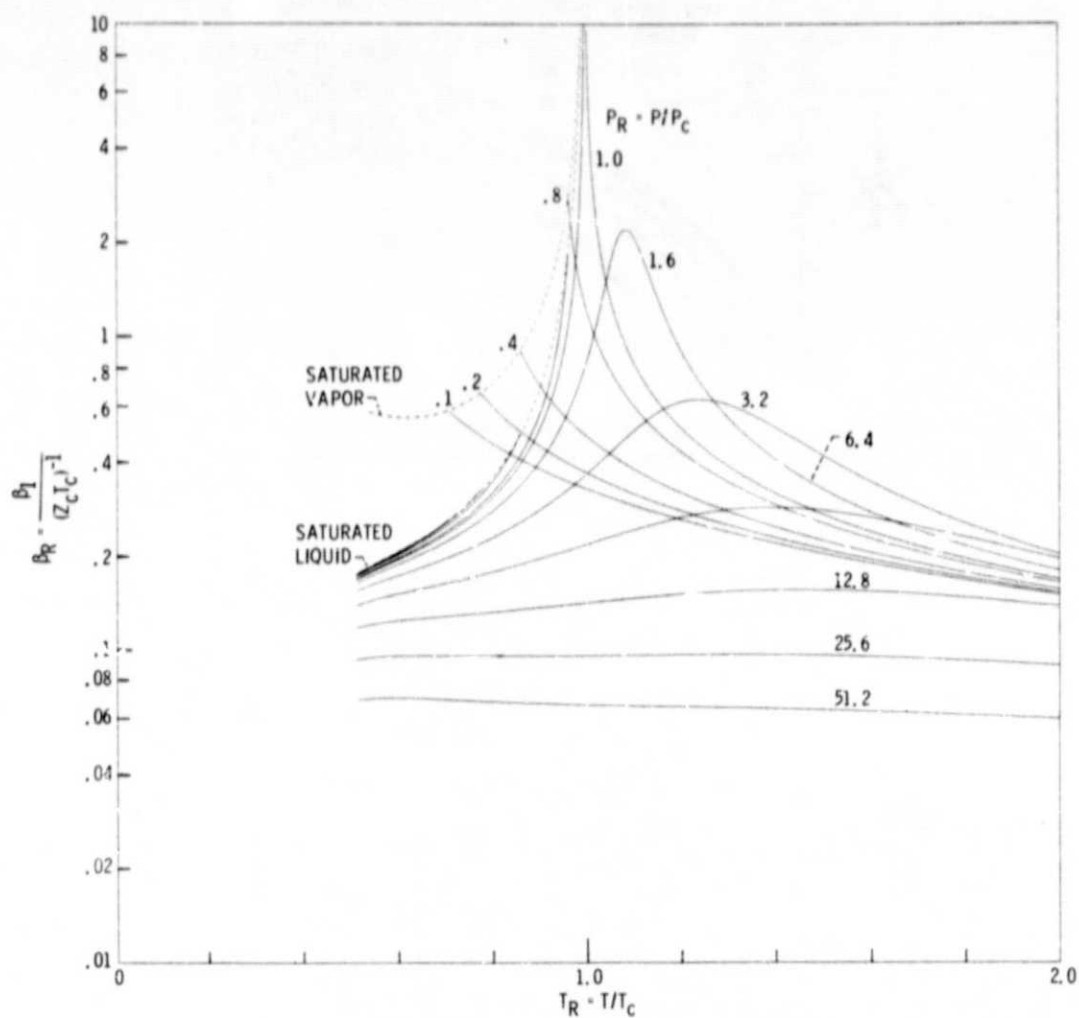


Fig. 10. Reduced volumetric expansion factor as a function of reduced temperature at selected reduced isobars, Hendricks (1979).

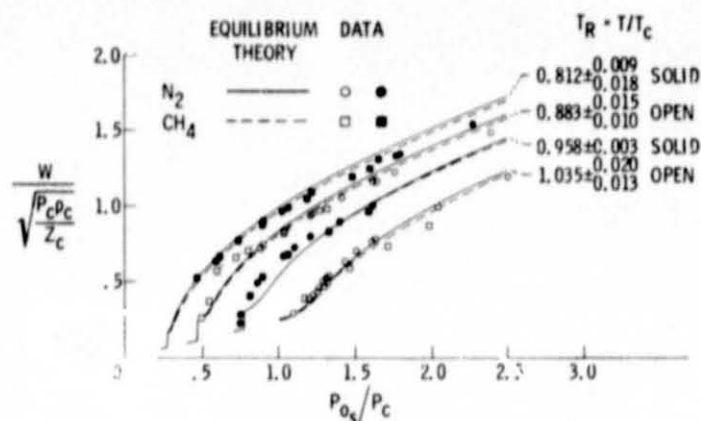


Fig. 11(a). Comparison of nitrogen and methane critical flow rate data on corresponding states basis, ref. 64.

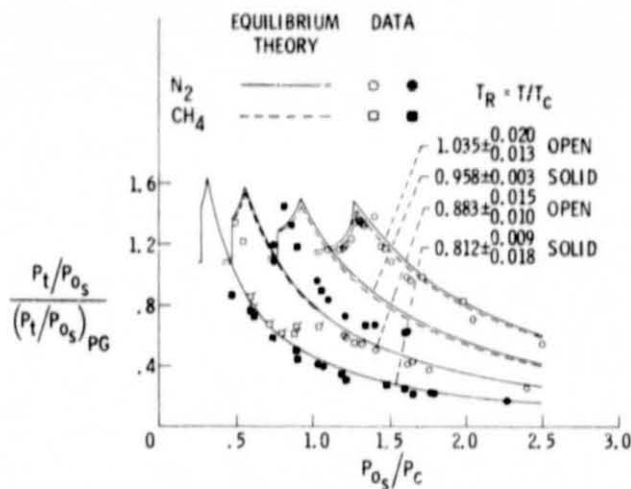


Fig. 11(b). Comparison of nitrogen and methane critical flow pressure ratio data on corresponding states basis, Hendricks and Simoneau (1973).

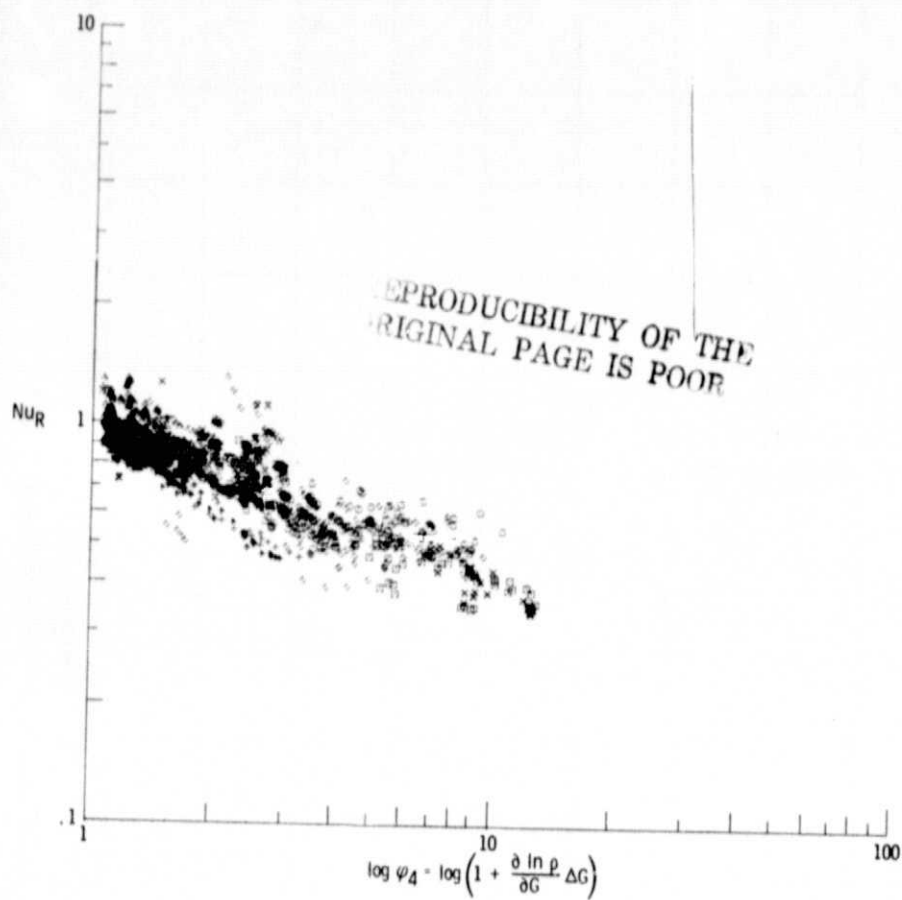
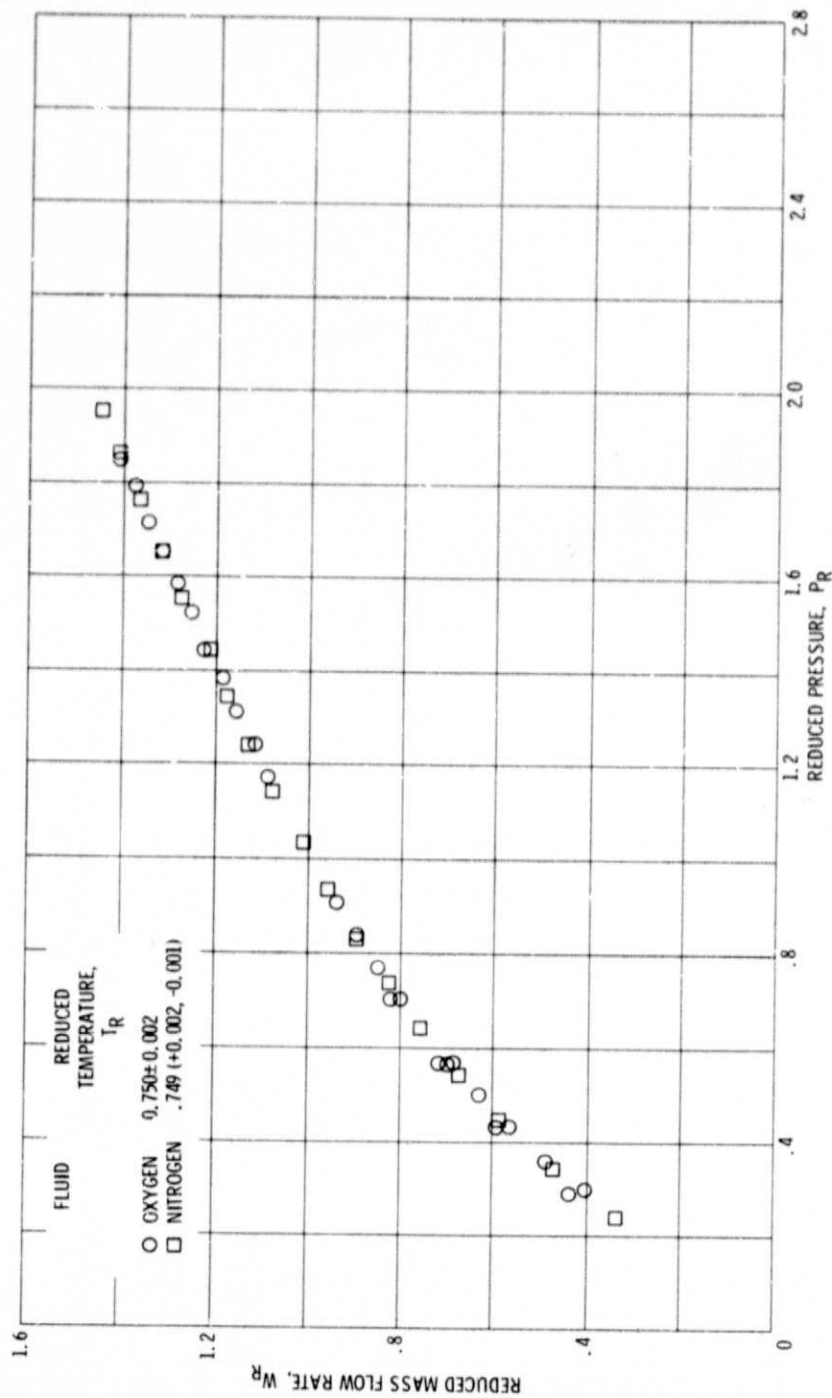


Fig. 12. Reduced Nusselt number as a function of Gibbs energy expansion factor for gaseous data from Hendricks (1979).



(a) CHOKED FLOW RATE FOR REDUCED TEMPERATURE OF -0.750.

Fig. 13. Choked flow rate and pressure ratio as function of reduced pressure and temperature - conical nozzle, Hendricks, Simoneau, and Barrows (1976).

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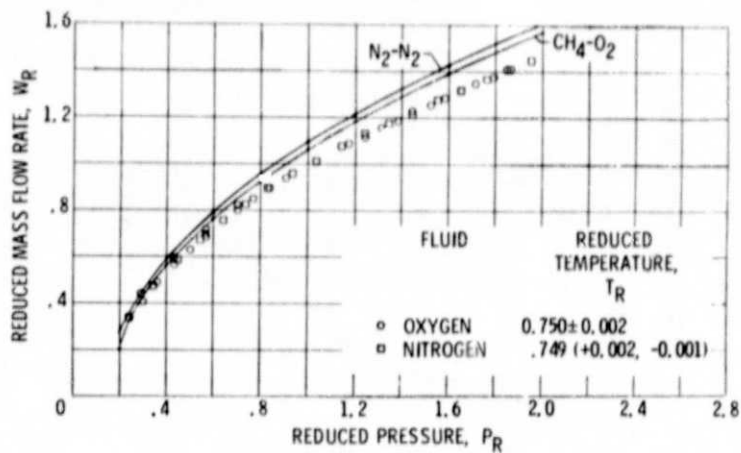


Fig. 14. Comparisons of choked flow rates as a function of reduced pressure for reduced temperature of ~ 0.750 .

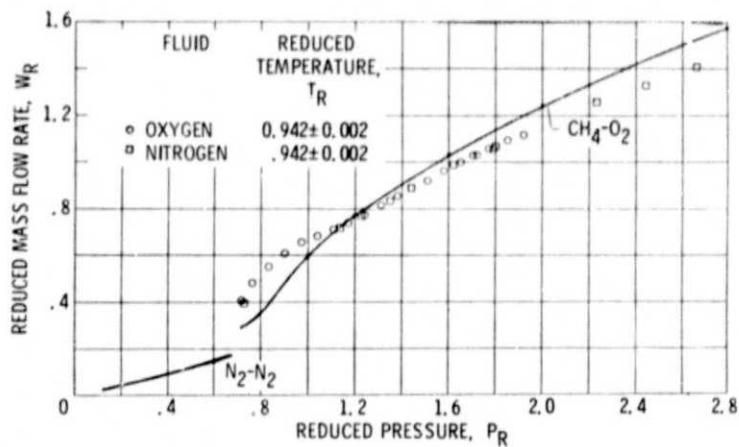


Fig. 15. Comparisons of choked flow rates as a function of reduced pressure for reduced temperature of ~ 0.942 .

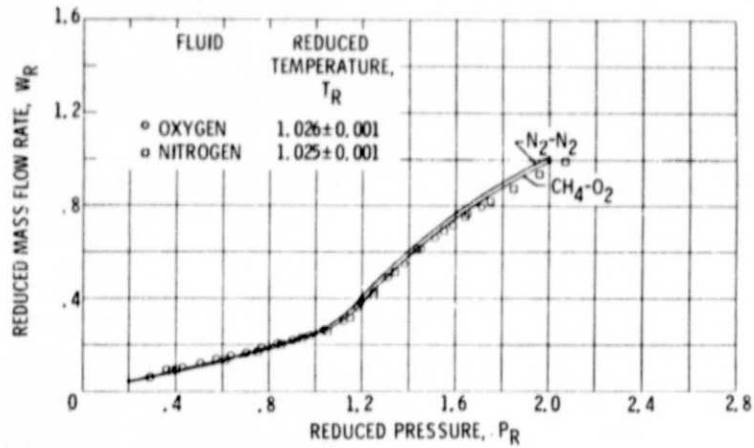


Fig. 16. Comparisons of choked flow rates as a function of reduced pressure for reduced temperature of ~ 1.025 .

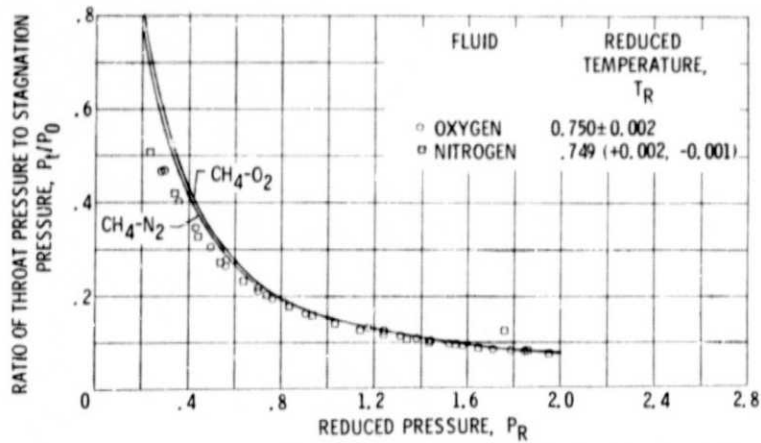


Fig. 17. Comparison of pressure ratio as a function of reduced pressure for reduced temperature of ~ 0.750 .

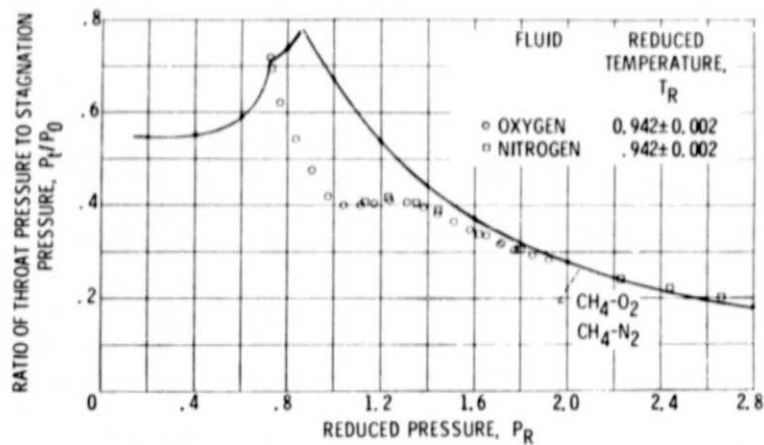


Fig. 18. Comparison of pressure ratio as a function of reduced pressure for reduced temperature of ~ 0.942 .

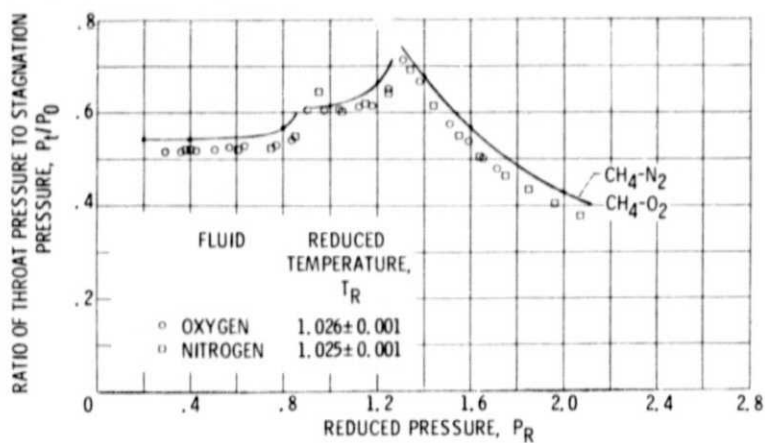


Fig. 19. Comparison of pressure ratio as a function of reduced pressure for reduced temperature of ~ 1.025 .

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